

INTRODUCTION TO

FLUID MECHANICS

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CHAPTER ONE

FLUID PROPERTIES AND DEFINITIONS

1.1 INTRODUCTION

Fluid Mechanics is a branch of applied mechanics concerned mainly with the study of the behaviour of fluids either at rest or in motion. The same ideas of momentum, energy, etc. used in ordinary mechanics can be applied, but frequently fluid mechanics is concerned with streams of fluid instead of individual bodies or particles.

Fluid

By fluids we mean either gaseous or liquid substances which are capable of flowing and conforming with the shape of containers. *A fluid is a material substance, which cannot sustain shear stress when it is at rest. In other words, a fluid is a substance, which deforms continuously under the action of shearing forces, however small they may be.*

Fluid may be divided into liquids and gases, the major differences between liquids and gases are:

- (a) Liquids are practically incompressible whereas gases are compressible and
- (b) Liquids occupy definite volumes and have free surfaces whereas a given mass of gas expands until it occupies all portions of any containing vessel.

Distinctions between a solid and a fluid

- (i) Up to the limit of elasticity the deformation of a solid is such that the strain is proportional to the applied stress, for a fluid the rate of strain is proportional to the stress.
- (ii) The strain of a solid is independent of the time of application of the force and if the elastic limit is not exceeded the deformation disappears when the stress is removed. But a fluid continues to flow as long as the stress is applied and does not recover its original form when the stress is removed.

1.2 PROPERTIES OF FLUIDS

In the development of the principles of fluid mechanics, some fluid properties play principal roles, others only minor roles or no roles at all. In fluid statics, weight is the important property, whereas in fluid flow, density and viscosity are predominant properties. Where appreciable compressibility occurs, principles of thermodynamics must be considered. Vapour pressure becomes important when negative pressures are involved, and surface tension affects static and flow conditions in small passages.

1.2.1 Density

The density or mass density of the fluid, ρ is defined as the *mass per unit volume*. Its unit of measurement is kg/m^3 i.e. $\rho = m/V$. For example, the density of water is 1000kgm^{-3} at 4°C . The density of gases can be derived from the equation of state for gases i.e. $PV = mRT$, where P = Absolute pressure (Pascal or Nm^{-2}); T = Absolute Temperature (K); R = Gas constant ($\text{Jkg}^{-1}\text{K}^{-1}$) and V = Volume (m^3). But $P = (m/V)RT$ or $P = \rho RT$. Therefore,

$$\rho = \frac{m}{V} \quad (1.1)$$

where ρ is the mass density.

1.2.2 Specific Volume

In addition to the mass density, the quantity specific volume ' V_s ' is sometimes used, being defined as the reciprocal of mass density, that is, specific volume is defined as *volume per unit mass*.

$$V_s = \frac{V}{m} = \frac{1}{\rho} = \frac{RT}{P} \quad (\text{m}^3\text{kg}^{-1}) \quad (1.2)$$

Example 1.1

What is the density of oxygen at 20°C under a pressure of 1.013 bar given that the gas constant is $260.1 \text{Jkg}^{-1}\cdot\text{K}^{-1}$?

Solution

$$T = 20 + 273 = 293 \text{ K}, \quad 1 \text{ bar} = 10^5 \text{ Nm}^{-2}$$

$$\rho = \frac{P}{RT}$$

$$\rho = \frac{1.013 \times 10^5}{260.1 \times 293} = \mathbf{1.329 \text{ kgm}^{-3}}$$

1.2.3 Specific Weight:

The specific weight ' Y ', of a fluid is its *weight per unit volume*. Its unit of measurement is Nm^{-3} .

$$Y = mg/V = \rho g \quad (1.3)$$

Where g = acceleration due to gravity = 9.81 ms^{-2}

Example 1.2

Find the specific weight of water at 4°C .

Solution

$$Y = \rho g = 1000 \times 9.81 = 9810 \text{ Nm}^{-3}.$$

1.2.4 Relative Density or Specific Gravity

The relative density RD of a substance is the ratio of *specific weight of the substance to the specific weight of water*, or *density of the substance to the density of water*, or *mass of the substance to the mass of equal volume of water* at specified temperature and pressure.

$$RD = \frac{Y_s}{Y_w} = \frac{\rho_s}{\rho_w} = \frac{m_s}{m_w} \quad (1.4)$$

where m_s = mass of substance and m_w = mass of equal volume of water.

Example 1.3

Find the density of oil whose relative density is 0.75.

Solution

$$RD = \rho_{oil}/\rho_w$$

$$\therefore \rho_{oil} = RD \times \rho_w$$

$$\rho_{oil} = 0.75 \times 1000 = 750 \text{kgm}^{-3}$$

1.3 COMPRESSIBILITY OF FLUIDS

The compressibility of a fluid is defined as a measure of change in volume of the fluid when it is subjected to the outside forces. All matter may be compressed to some extent by the application of pressure and strain energy is being stored as a result of change in volume in the process. The compressibility of any substance is measure in terms of *bulk modulus of elasticity*, K.

1.3.1 Bulk Modulus of Elasticity

Also known as Modulus of volume expansion is defined as *the ratio of the change in pressure to the corresponding volumetric strain*.

$$\text{Bulk Modulus} = \frac{\text{Change in Pressure}}{\text{Volumetric Strain}}$$

$$K = \frac{-dP}{\frac{dV}{V}} \quad (\text{Nm}^{-2}) \quad (1.5)$$

which means that K is a measure of the incremental change in pressure dP which takes place when a volume V is changed on incremental amount dV (Fig. 1.1). *Volumetric strain is the change in volume divided by the original volume*. The negative sign implies a decrease in volume arising from an increase in pressure. Since there is a change in volume, there must be a change in density of the substance; and K can be express in terms of density ρ from

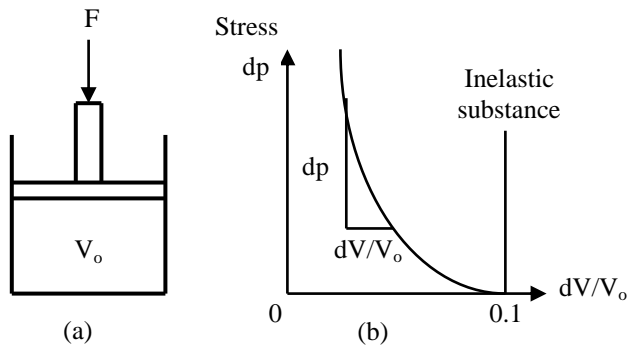


Fig. 1.1 Compressibility of fluids

$$\rho = m/V$$

Differentiating w.r.t. V

$$\frac{d\rho}{dV} = -\frac{m}{V^2}$$

$$\frac{d\rho}{dV} = -\frac{\rho V}{V^2} = -\frac{\rho}{V}$$

$$\frac{d\rho}{\rho} = -\frac{dV}{V} \quad \text{or} \quad \frac{dV}{V} = -\frac{d\rho}{\rho}$$

substituting in Eq. (1.5) above, we have

$$K = \rho \left(\frac{dP}{d\rho} \right) \quad (1.6)$$

The reciprocal of the bulk modulus of elasticity is termed compressibility. For liquids, the bulk moduli are very high meaning that the compressibility is very low. For most practical purposes, liquids may be regarded as incompressible. However, there are cases such as steady flow in pipes where compressibility should be taken into consideration. Since volumetric strain is the ratio of two volumes, the units of bulk modulus will be the same as those of pressure, Newtons per square metre (Nm^{-2}).

Example 1.4

- (a) Find the change in volume of 1.00 m^3 of water at 26.7°C when subjected to a pressure of 2.0 bar and bulk modulus of elasticity is 2.24 GNm^{-2}
- (b) From the following test data, determine the bulk modulus of elasticity of water at 35 bar, the volume was 1.00 m^3 and at 240 bar, the volume was 0.99 m^3

Solution

$$(a) \quad K = \frac{-dP}{\frac{dV}{V}} \quad (\text{Nm}^{-2})$$

$$dV = -V \left(\frac{dP}{K} \right)$$

$$dV = -1.00 \left(\frac{2 \times 10^5}{2.24 \times 10^9} \right) \\ = -0.000089 \text{ m}^3 \text{ or } -8.9 \times 10^{-5} \text{ m}^3$$

$$(b) \quad dP = 240 - 35 = 205 \text{ bar}$$

$$dV = V_2 - V_1 = 0.99 - 1.00 = -0.01 \text{ m}^3$$

$$K = -V \left(\frac{dP}{dV} \right)$$

$$K = -1.00 \left(\frac{205 \times 10^5}{-0.01} \right) \\ = 2.05 \times 10^9 \text{ Nm}^{-2} = \mathbf{2.05 \text{GNm}^{-2}}$$

1.3.2 Compression of Gases

The compression and expansion of gases takes place according to various laws of thermodynamics.

Isothermal process: is a constant temperature process and is characterised by Boyle's law.

$$PV = \text{Constant}$$

$$P_1V_1 = P_2V_2$$

The bulk modulus can be derived as follows:

By Definition,

$$K = -V \left(\frac{dP}{dV} \right) \quad (i)$$

$$\text{and } PV = C \quad (ii)$$

$$P = CV^{-1} \quad (iii)$$

Differentiating (iii) w.r.t. 'V' we have

$$\frac{dP}{dV} = -\frac{C}{V^2} \quad (iv)$$

substituting (iv) in (i), we have

$$K = -V(-CV^{-2}) = CV^{-1} \quad (v)$$

substituting (ii) in (v), we have

$$K = (PV)V^{-1} \text{ or } K = P \quad (1.7)$$

Hence, for an isothermal process, the pressure and the bulk modulus of elasticity are the same.

Isentropic process: Isentropic or reversible adiabatic process is a frictionless process in which no heat exchange across the boundary. This follows the relation

$$PV^\gamma = \text{Constant}$$

where, γ = adiabatic index, which is the ratio of two specific heats, that is, $\gamma = c_p/c_v$

The bulk modulus can be derived as follows:

By Definition,

$$K = -V \left(\frac{dP}{dV} \right) \quad (\text{i})$$

$$PV^\gamma = C \quad (\text{ii})$$

$$P = CV^{-\gamma} \quad (\text{iii})$$

Differentiating w.r.t. 'V'

$$\frac{dP}{dV} = -C\gamma(V^{-\gamma-1}) \quad (\text{iv})$$

Substituting (ii) in (iv), we have

$$\frac{dP}{dV} = -\gamma PV^\gamma(V^{-\gamma-1}) = -\gamma PV^{-1} \quad (\text{v})$$

Substituting (v) in (i), we have

$$K = -V(-\gamma PV^{-1})$$

$$K = P\gamma \quad (1.8)$$

Example 1.5

A cylinder contains 0.48 m³ of air at 50 °C and 2.8 bar. The air is compressed to 0.096 m³.

- Assuming isothermal conditions, what is the pressure at the new volume and what is the bulk modulus of elasticity?
- Assuming isentropic conditions, what is the final pressure and temp, and what is the bulk modulus of elasticity? Given that $\gamma = 1.4$

Solution

(a) For isothermal:

$$P_1V_1 = P_2V_2$$

$$\therefore P_2 = \frac{P_1V_1}{V_2} = \frac{2.8 \times 10^5 \times 0.48}{0.096}$$

$$= 1400000 \text{ Nm}^{-2} = 14 \text{ bar}$$

for isothermal $K = P_2 \therefore K = 14 \text{ bar}$

(b) For isentropic:

(i) The final pressure:

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\therefore P_2 = P_1 \left(\frac{V_1}{V_2} \right)^\gamma = 2.8 \left(\frac{0.48}{0.096} \right)^{1.4}$$

$$= 2.8 \times (5)^{1.4} = \mathbf{26.65 \text{ bar}}$$

(ii) The final temperature

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore T_2 = 323 \left(\frac{26.65}{2.8} \right)^{0.4}$$

$$= 323 \times 1.904 = 615 \text{ K or } \mathbf{342 \text{ }^\circ\text{C}}$$

(iii) $K = P_2 \cdot \gamma = 26.65 \times 10^5 \times 1.4$

$$K = 37.31 \times 10^5 \text{ N/m}^2 \text{ or } \mathbf{3.73 \text{ MNm}^{-2}}$$

1.4 VISCOSITY OF FLUIDS

The viscosity of a fluid is that property which determines its ability to resist shearing stress or angular deformation. A fluid at rest offers no resistance to shearing forces but once it is in motion shearing forces are set up between layers of fluid moving at different velocities. Viscosity is due fundamentally to cohesion and molecules momentum exchange between fluid layers as flow occurs.

1.4.1 Expression for Dynamic Viscosity

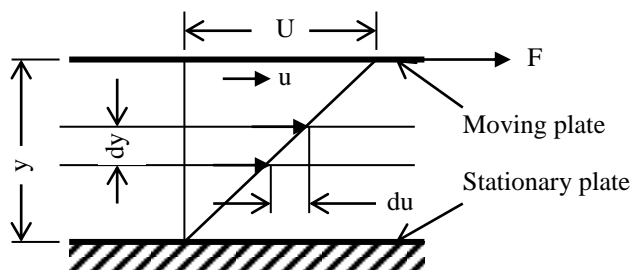


Fig. 1.2: Shearing in a fluid

Consider the motion of a high viscous fluid illustrated in Fig. 1.2, the fluid is confined between two parallel plates by a small distance y . The lower plate is stationary while the upper plate is moving with velocity U . The upper plate is acted upon by a constant force F and thus moves at a constant velocity U . Because of adhesion, the fluid in contact with the upper plate moves with the same velocity as the plate, whereas the fluid in contact with the stationary plate has a velocity equal to zero. A velocity gradient is thus set up in the fluid. Experiments have shown that the force F varies with the area of the plate A , with velocity U and inversely with distance y .

$$F \propto A \frac{du}{dy}$$

where, du/dy = velocity gradient

$$\therefore \frac{F}{A} \propto \frac{du}{dy}$$

But the shear stress, τ , is defined as $\tau = F/A$

$$\text{Hence, } \tau \propto \frac{du}{dy}$$

$$\therefore \tau = \mu \left(\frac{du}{dy} \right) \quad (1.9)$$

where μ is the constant of proportionality called **Absolute** or **Dynamic viscosity**. This was discovered by Newton. The coefficient of Dynamic viscosity, μ is defined as the shear force per unit area required to draw one layer of fluid with unity velocity past another layer unit distance away from it in the fluid. Its unit is Ns/m^2 or kg/ms

$$\mu = \left(\frac{\tau}{du/dy} \right) = \frac{\text{Force/Area}}{\text{Velocity/Distance}}$$

$$\mu = \frac{\text{Force} \times \text{Time}}{\text{Area}} \quad (1.10)$$

1.4.2 Kinematic Viscosity

Kinematic viscosity, ν is defined as the ratio of dynamic viscosity to mass density. Unit is m^2s^{-1}

$$\nu = \mu / \rho \quad (1.11)$$

1.4.3 Newtonian and Non – Newtonian Fluids

(a) **Ideal Fluid:** For the ideal fluid, the resistance to shearing deformation is zero, and hence the plotting coincides with the x-axis.

(b) **Ideal or Elastic Solid:** For the ideal or elastic solid, no deformation will occur under any loading condition, and the plotting coincides with y-axis.

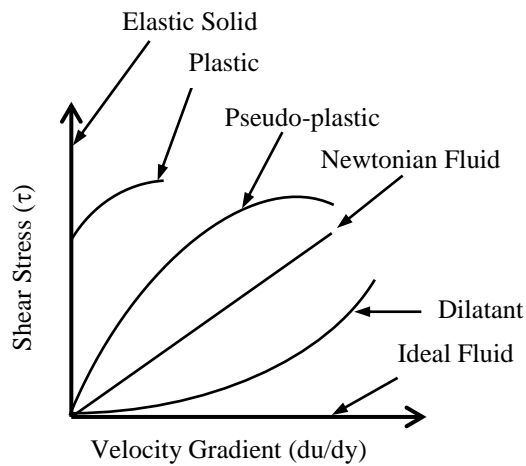


Fig. 1.3: Shearing stress versus shearing strain for various types of fluids

(c) **Newtonian Fluids:** Fluids obeying Newton's law of viscosity and for which μ has a constant value are known as Newtonian fluids. Most common fluid falls into this category, for which shear stress is linearly related to velocity gradient.

(d) **Non-Newtonian Fluids:** Fluids which do not obey Newton's law of viscosity are known as non-Newtonian fluid and they fall into one of the following groups:

(i) *Plastic:* For which the shear stress must reach a certain minimum value before flow commences. Thereafter, shear stress increases with the rate of shear according to the relationship.

$$\tau = A + B(du/dy)^n \quad (1.12)$$

Where A, B and n are constants e.g. sewage sludge

(ii) *Pseudo-plastic:* For which dynamic viscosity decreases as the rate of shear increases (e.g. colloidal solutions, milk, and cement).

(iii) *Dilatant substances,* in which dynamic viscosity increases as the rate of shear increases (e.g. quick sand)

(iv) *Thixotropic substances,* for which the dynamic viscosity decreases with the time for which shearing forces are applied (e.g. thixotropic jelly).

(v) *Rheopectic materials,* for which the dynamic viscosity increases with the time for which shearing forces are applied.

(vi) *Viscoelastic materials,* which behave in a manner similar to Newtonian fluids under time-variant conditions but, if the shear stress changes suddenly, behave as if plastic.

It must be noted that viscosities of liquid decreases with temperature increase, while the absolute viscosity of gases increases with increase in temperature. The absolute viscosity of liquids is not appreciably changed by change in pressure. Since the density of gases changes with pressure changes at constant temperature, the kinematic viscosity varies inversely as the pressure.

1.5 SURFACE TENSION.

The liquid surface behaves as if it were an elastic membrane under tension. The apparent tension effect, when the liquid is in contact with another fluid or solid, depends upon the relative sizes of inter molecular cohesion and adhesion. The surface tension, σ , is defined as *the force in the liquid normal to a line of unit length drawn in the surface*. Its unit of measurement is N/m.

Considering the forces acting on a diametral plane through a spherical drop of radius r ,

$$\text{Force due to internal pressure} = P \times \pi r^2$$

$$\text{Force due to surface tension around the perimeter} = 2\pi r \times \sigma$$

For equilibrium

$$P\pi r^2 = 2\pi r\sigma \text{ or } P = 2\sigma/r \quad (1.13)$$

The effect of surface tension is called *surface energy* and is expressed in $\text{N}\cdot\text{m}/\text{m}^2$ or J/m^2 . In this case surface tension (σ) represents the stretching work that needs to be done to increase the surface area of the liquid by a unit amount. The surface tensions of some fluids are listed in Table 1.1.

Table 1.1: Surface tensions of some fluids in air at 1 atmosphere.

Fluid	Surface Tension, σ (Nm^{-1})
Water at 0°C	0.076
at 20°C	0.073
at 100°C	0.059
at 300°C	0.014
Glycerin at 20°C	0.063
SAE 30 oil at 20°C	0.035
Mercury at 20°C	0.440
Ethyl alcohol at 20°C	0.023
Blood at 37°C	0.058
Gasoline at 20°C	0.022
Ammonia at 20°C	0.021
Soap solution at 20°C	0.025
Kerosene at 20°C	0.028

In many engineering problems, the effect of surface tension is usually ignored because of its magnitude is relatively small compared with the other forces acting on the fluid. However these forces can cause serious errors in hydraulic scale models and through capillary effects. Surface tension forces can be reduced by the addition of detergents.

1.6 CAPILLARITY

Another interesting consequence of surface tension is the *capillary effect*, which is the rise and fall of a liquid in a small-diameter tube inserted into the liquid. Such narrow tubes or confined flow channels are called *capillaries*. The rise of kerosene through a cotton wick inserted into the reservoir of a kerosene lamp is due to capillary effect.

It is commonly observed that water in a glass container curves up slightly at the edges where it touches the glass surface; but the opposite occurs for mercury: it curves down at the edges. This effect is usually expressed by saying that water wets the glass (by sticking to it) while mercury does not. The strength of the capillary effect is quantified by the contact angle θ , define as the angle that the tangent to the liquid surface makes with the solid surface at the point of contact. The surface tension force acts along this tangent line toward the solid surface.

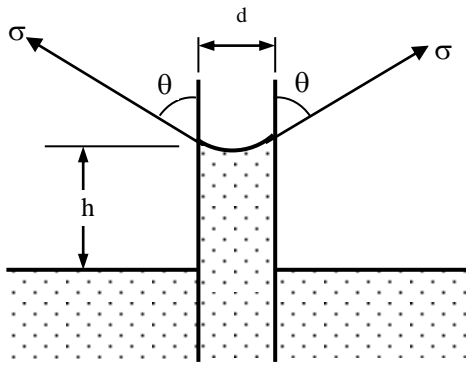


Fig. 1.4: Capillarity

Consider a fine tube of diameter 'd' as shown in the Fig. 1.4. The magnitude of the capillary rise in a circular tube can be determined from a force balance on the cylindrical liquid column of height h in the tube. Let θ be the angle of contact between the liquid and the solid. The upward pull due to surface tension is

$$F_{v_1} = \sigma \pi d \cos \theta$$

The weight of the liquid raised, $F_{v_2} = \rho g h (\frac{1}{4}\pi d^2)$

Under equilibrium condition, $F_{v_1} = F_{v_2}$

$$\sigma \pi d \cos \theta = \rho g h (\frac{1}{4}\pi d^2)$$

$$\text{Hence, } h = \frac{4\sigma \cos \theta}{\rho g d} \quad (1.14)$$

where, h = height of liquid rise;

d = diameter of tube;

ρ = density of liquid;

g = acceleration due to gravity.

For non-wetting liquids (such as mercury in glass) θ is greater than 90° , which makes negative. Therefore, a negative value of capillary rise corresponds to capillary drop. This relation (Equation 1.14) shows that capillary rise is inversely proportional to the diameter of the tube. Therefore, the thinner the tube is the greater the rise (or fall) of the liquid will be in the tube. In practice, the capillary effect is usually negligible in tubes whose diameter is greater than 1 cm. The capillary rise is also inversely proportional to the density of the liquid. Therefore, lighter liquids will experience greater capillary rise.

1.7 VAPOUR PRESSURE

Since the molecules of a liquid are in constant agitation, of the molecules in the surface layer will have sufficient energy to escape from the attraction of the surrounding molecules into the space above the free surface. Some of these molecules will return and condense, but others will take their place. If the space above the liquid is confined, equilibrium will be reached so that the number of molecules of liquid in the space above the free surface is constant. These molecules produce a partial pressure known as the *vapour pressure* in the space.

An increase in temperature is associated with a greater molecular agitation and therefore increases the vapour pressure. Boiling will occur when the vapour pressure is equal to the pressure above the liquid. By reducing the pressure, boiling can be made to occur at temperatures well below the boiling point at atmospheric pressure.

1.8 CAVITATION

It is quite possible, under certain condition for areas of low pressure to occur locally in a flowing fluid. In such areas, the variation of pressure becomes very important. If the vapour in such an area falls below the vapour pressure, there will be local boiling and a cloud of vapour bubbles will form. This phenomenon is known as *cavitation* and can cause serious problems, since the flow of liquid can sweep this cloud of bubbles on into an area of higher pressure where the bubbles will collapse suddenly. If this

should occur in contact with a solid surface, very serious damage can result due to the very large force with which the liquid hits the surface.

Cavitation can also occur, if a liquid contains dissolved air or other gases, since the solubility of gases in a liquid decreases as the pressure is reduced. Gas or air bubbles will be released in the same way as vapour bubbles, with the same damaging effect. Usually, this release occurs at higher pressures and therefore, before vapour cavitation commences. Cavitation is a serious problem in the field of hydraulic machinery as it has a significant effect on the performances of pumps, turbines and propellers, and also, the impact of collapsing bubbles can cause local erosion of metal surfaces.

1.9 FLUID PRESSURE

Pressure is expressed as the *force per unit area*.

$$P = F/A. \quad (\text{Nm}^{-2}) \quad (1.15)$$

There are different types of pressure encountered in fluid mechanics.

- (a) **Atmospheric pressure:** This is the pressure due to the atmosphere at the earth surface as measured by a barometer. The pressure decreases with altitude e.g. at mean sea level (m.s.l), the pressure is $1.01325 \times 10^5 \text{ Nm}^{-2}$.
- (b) **Gauge pressure:** This is the intensity of pressure measured above or below the atmospheric pressure. It is measured by manometer,
- (c) **Absolute pressure:** This is the summation of Gauge and atmospheric pressure.
- (d) **Vacuum:** A Perfect vacuum is a completely empty space, therefore, the pressure is zero.

1.8.1 Pressure Disturbances

Pressure disturbances imposed on a fluid moves in waves. This pressure waves move at a velocity equal to that of sound through the fluid. The velocity or celerity (quickness) is expressed as $c = \sqrt{K/\rho}$. For

gases, this acoustic velocity is $c = \sqrt{\gamma P/\rho} = \sqrt{\gamma RT}$, since $P/\rho = RT$

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CHAPTER TWO

FLUID STATICS

Fluid statics or hydrostatics is the study of force and pressure in a fluid at rest with no relative motion between fluid layers. From the definition of a fluid, there will be no shearing forces acting and therefore, all forces exerted between the fluid and a solid boundary must act at right angles to the boundary. If the boundary is curved, it can be considered to be composed of a series of chords on which a force acts perpendicular to the surface concerned.

2.1 PRESSURE

A fluid will exert a force normal to a solid boundary or any plane drawn through the fluid. In many cases the magnitude of the force exerted on a small area of the boundary or plane may vary from place to place, it is convenient to work in terms of the pressure of the fluid. Pressure at any point is *defined as the force per unit area* at that point.

$$P = \frac{\text{force}}{\text{area}} = \frac{F}{A} \text{ (Nm}^{-2}\text{)} \quad (2.1)$$

2.2 TRANSMISSION OF FLUID PRESSURE (PASCAL'S LAW)

The principle of transmission of fluid pressure states that *the pressure intensity at any point in a fluid at rest is transmitted without loss to all other points in the fluid*. The principle can be illustrated by a simple hydraulic jack (Fig. 2.1). If a force F is applied to the plunger and the piston C moves down through a distance x , piston D carrying a load of W will also move up through a distance y . The volume of liquid displaced at A

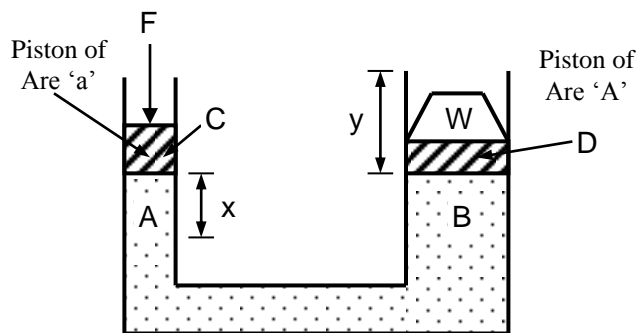


Fig. 2.1: Hydraulic jack

$$V_A = ax$$

And at B, $V_B = Ay$

Since there is no loss of fluid

$$V_A = V_B \text{ and } ax = Ay$$

$$y = \frac{ax}{A} \quad (2.2)$$

The work done at A, $W_A = Fx$

and Similarly at B, $W_B = Wy$

$$W_A = W_B \text{ and } Fx = Wy \quad (2.3)$$

$$Fx = W \left(\frac{ax}{A} \right) \text{ since } y = ax/A$$

$$\text{Therefore, } \frac{F}{a} = \frac{W}{A} \quad (2.4)$$

$$\text{i.e. } P_A = P_B$$

This indicates that the pressure at a point in a fluid at rest is the same in all direction.

Example 2.1

A force F of 850 N is applied to the smaller cylinder of an hydraulic jack. The area of the small piston is 15 cm^2 and that of the larger piston is 150 cm^2 . What load W can be lifted on the larger piston, if the pistons are at the same level. The density of the liquid in the jack is 1000 kgm^{-3}

Solution

$$\frac{F}{a} = \frac{W}{A}$$

$$W = F \left(\frac{A}{a} \right)$$

$$W = 850 \left(\frac{150}{15} \right) = 8500 \text{ N}$$

$$\text{Mass lifted} = W/g = 8500/9.81 = 868 \text{ kg.}$$

2.3 PRESSURE IN A FLUID DUE TO ITS OWN WEIGHT

2.3.1 Fluids of Uniform Density

Consider a vertical tube of fluid of height h , uniform cross-sectional area A , specific weight Y and density. The total weight of fluid is $W = mg$

$$\text{But } m = \rho v \text{ where } V = Ah$$

Therefore, $W = \rho gAh$ (2.5)

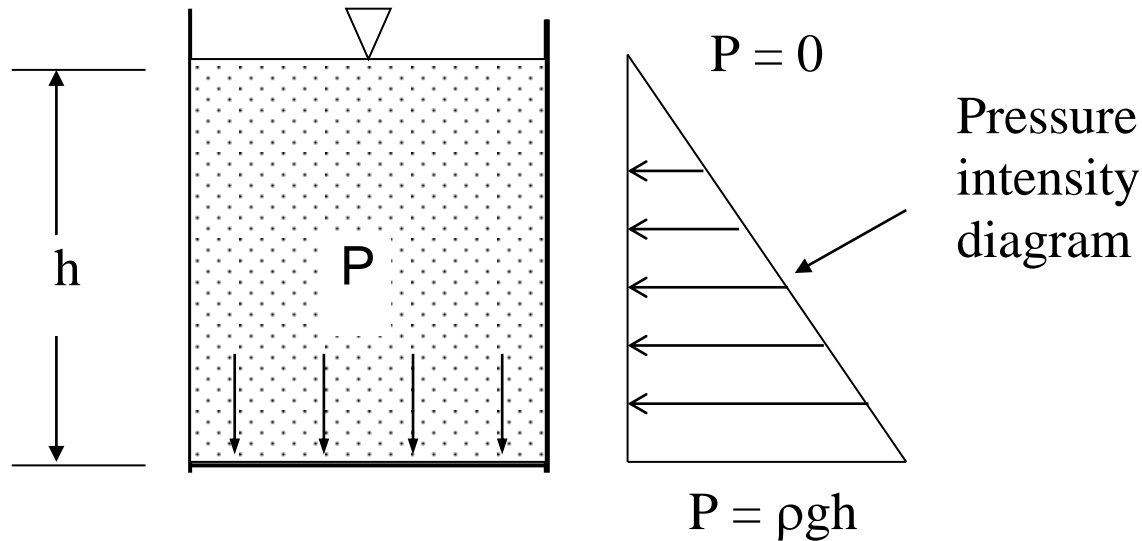


Fig. 2.2: Pressure due to fluids of uniform density

Pressure 'P' at a depth 'h' is expressed as

$$P = \text{Weight of fluid}/\text{Area} = \rho gAh/A$$

$$P = \rho gh \quad (2.6)$$

If the atmospheric pressure is P_a then the absolute pressure or total pressure at depth h in the liquid is

$$P = P_a + \rho gh \quad (2.7)$$

Example 2.2

Determine the pressure at a depth of 6 m in oil of relative density 0.75.

Solution

$$\text{Density of oil} = 0.75 \times 1000 = 750 \text{ kgm}^{-3}$$

$$\begin{aligned} \text{Pressure, } P &= \rho gh = 750 \times 9.81 \times 6 \\ &= 44145 \text{ Nm}^{-2} \text{ or } 0.44151 \text{ bar} \end{aligned}$$

2.3.2 Stratified Fluids

Hydrostatic pressure is caused simply by the weight of the liquid above the point (or surface) being considered. Stratified fluids are two or more fluids of different densities, which float on the top of one another without mixing together. The total weight (or pressure) exerted by the fluids can be obtained by first calculating the weight (or pressure) of individual fluid and then add them together. Consider a rectangular tank in Fig. 2.3. Let the base area be $A \text{ m}^2$, the pressure P_1 , and weight W_1 of the fluid at the top of the tank is given by:

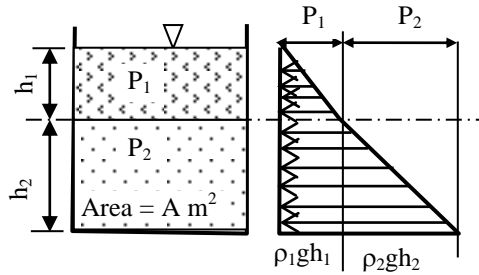


Fig. 2.3: Stratified fluids

$$P_1 = \rho_1gh_1 \text{ and } W_1 = \rho_1gh_1A.$$

Similarly, the pressure P_2 , and weight W_2 of the fluid at the bottom of the tank is:

$$P_2 = \rho_2gh_2 \text{ and } W_2 = \rho_2gh_2A.$$

$$\text{Total pressure } P_T = P_1 + P_2 = \rho_1gh_1 + \rho_2gh_2 \quad (2.8)$$

$$\text{Total weight, } W_T = W_1 + W_2$$

$$W_T = \rho_1gh_1A + \rho_2gh_2A = (\rho_1gh_1 + \rho_2gh_2)A$$

\therefore Total weight = Total pressure x Area

$$W_T = P_TA \quad (2.9)$$

2.4 MEASUREMENT OF ABSOLUTE PRESSURE

The absolute pressure of a liquid is measured by a *barometer*. Barometer consists of a tube sealed at the top and standing with its open end in mercury (Fig. 2.4). If the mercury rise in the tube is h , by the principle of transmissibility of pressure the upward pressure exerted by the mercury in the tube at A is equal to the downward pressure P_a .

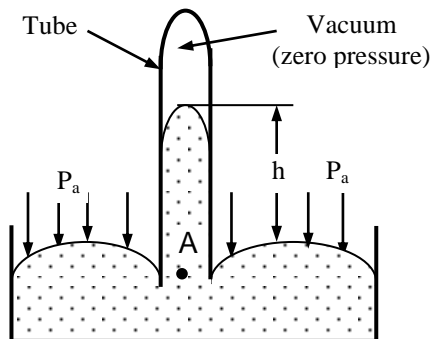


Fig. 2.4: The mercury barometer

Pressure at A due to the weight of the mercury is given by

$$P = \rho gh = P_a, \quad h = P_a / \rho g \quad (2.10)$$

2.5 PRESSURE MEASUREMENT BY MANOMETER

The relationship between pressure and head is utilized for pressure measurement in the manometer or liquid gauge.

2.5.1 Piezometer Tube

The simplest form is the pressure tube or piezometer shown in Fig. 2.5. It consists of a single vertical tube, open at the top, inserted into a pipe or vessel containing liquid under pressure which rises in the tube to a height depending on the pressure. If the top of the tube is open to the atmosphere, the pressure measured is “gauge” pressure.

Pressure at A = Pressure due to column of liquid of height h_1

$$P_A = \rho gh_1 \quad (2.11)$$

Similarly, $P_B = \rho gh_2$ (2.12)

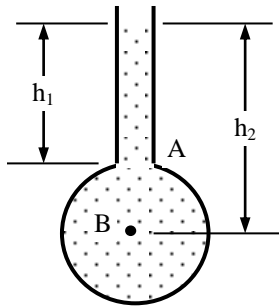


Fig. 2.5: Piezometer

Example 2.3

A pressure tube is used to measure the pressure of oil of density 850 kgm^{-3} in a pipeline. If the oil rises, to a height of 0.8 m above the centre of the pipe, determine the gauge pressure in Nm^{-2} at the centre of the pipe.

Solution

$$P = \rho gh = 850 \times 9.81 \times 0.8 = \mathbf{6670.8 \text{ Nm}^{-2}}$$

2.5.2 Open-End U-Tube Manometer

The U-tube gauge shown in the Fig. 2.6 below can be used to measure the pressure of either liquids or gases.

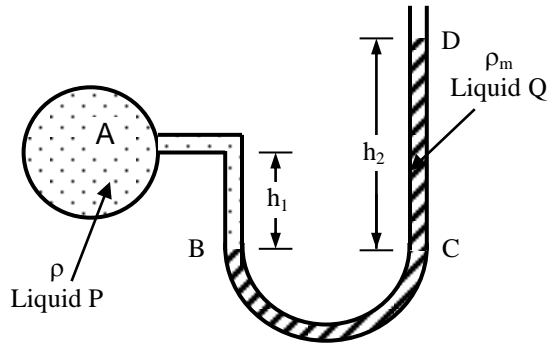


Fig. 2.6: Open-end U-tube manometer

The bottom of the U-tube is filled with a manometric liquid Q, which is of greater density ρ_m and is immiscible with the fluid P, liquid or gas, of density ρ , whose pressure is to be measured. If B is the level of the interface in the left-hand limb and C is a point at the same level in the right-hand limb,

$$\text{pressure } P_B = \text{Pressure } P_C$$

But $P_B = \text{Pressure } P_A + \text{Pressure due to depth } h_1 \text{ of fluid P}$

$$= P_A + \rho g h_1$$

$$P_C = \text{Pressure } P_D + \text{Pressure due to depth } h_2 \text{ of liquid Q}$$

But $P_D = \text{Atmospheric pressure} = \text{Zero gauge pressure}$

$$\therefore P_C = 0 + \rho_m g h_2 \text{ Since } P_B = P_C$$

$$P_A + \rho g h_1 = \rho_m g h_2$$

$$P_A = \rho_m g h_2 - \rho g h_1 \quad (2.13)$$

Example 2.4

In Fig. 2.6 if liquid Q is mercury and is 30 cm below A in the left-hand limb and 20 cm above A in the right-hand limb, what is the gauge pressure at A? Relative density of mercury is 13.6 and the density of fluid in A is 800 kgm^{-3}

Solution

$$\text{Density of mercury} = 13.6 \times 1000 = 13600 \text{ kgm}^{-3}$$

$$h_1 = 30 \text{ cm} = 0.3 \text{ m}$$

$$\text{and } h_2 = 30 + 20 = 50 \text{ cm or } 0.5 \text{ m}$$

$$P_A = \rho_m g h_2 - \rho g h_1$$

$$P_A = (13600 \times 9.81 \times 0.5) - (800 \times 9.81 \times 0.3)$$

$$= 66708 - 2354.4$$

$$P_A = \mathbf{64354 \text{ Nm}^{-2}}$$

2.5.3 Close-End U - Tube Manometer:

The pressure difference between two points in a fluid can be measured either (a) by measuring the pressure at each point separately and subtracting or (b) by using a differential pressure gauge or manometer which measures pressure difference between two different points in the same pipeline or two different chambers

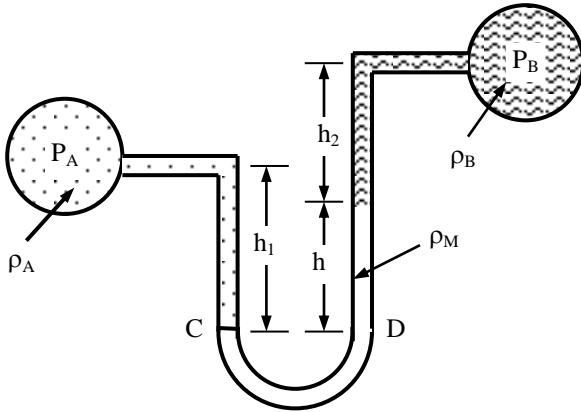


Fig. 2.7: Close-end U-tube manometer

As in the previous case, the principle involved in calculating the pressure difference is that the pressure at the same level CD in the two limbs must be the same, since the fluid in the bottom of the U-tube is at rest. Suppose P_A and P_B are the source pressures, ρ_A and ρ_B the fluid densities, ρ_m the manometer liquid density, and elevations h_1 , h_2 and h as shown in Fig. 2.7.

Equating the pressure in each limb at level CD

$$P_C = P_A + \rho_A g h_1 \quad \text{and}$$

$$P_D = P_B + \rho_B g h_2 + \rho_m g h$$

But $P_C = P_D$, hence,

$$P_A + \rho_A g h_1 = P_B + \rho_B g h_2 + \rho_m g h$$

$$P_A - P_B = \rho_B g h_2 + \rho_m g h - \rho_A g h_1 \quad (2.14)$$

Example 2.5

A U-tube manometer is arranged as shown in Fig. 2.7, to measure the pressure difference between two pipelines A and B, both carrying water of density 1000 kg/m^3 . The density of manometric fluid is 13600 kgm^{-3} . Calculate the pressure difference if h_1 is 1.2 m, h_2 is 0.8 m and h is 0.7 m.

Solution

$$P_A - P_B = \rho_B g h_2 + \rho_m g h - \rho_A g h_1$$

$$\text{Since } \rho_A = \rho_B = 1000 \text{ kgm}^{-3}$$

$$\therefore P_A - P_B = \rho_A g(h_2 - h_1) + \rho_m g h$$

$$P_A - P_B = 1000 \times 9.81(0.8 - 1.2) + 13600 \times 9.81 \times 0.7$$

$$P_A - P_B = 3924 + 93391.2 = \mathbf{89467.2 \text{ Nm}^{-2}}$$

2.5.4 Inverted U-Tube Manometer

The inverted U-tube manometer shown in Fig. 2.8 is used for measuring pressure differences in liquids. The top of the U-tube is filled with a fluid, frequently air, which is less dense than that connected to the instrument. Since the fluid in the top is at rest, pressures at level CD will be the same in both limbs.

$$P_A = \rho_A g h_1 + \rho_m g h + P_C \quad \text{and} \quad P_B = \rho_B g h_2 + P_D$$

$$\therefore P_A - P_B = \rho_A g h_1 + \rho_m g h + P_C - \rho_B g h_2 - P_D$$

Since $P_C = P_D$

$$\therefore P_A - P_B = \rho_A g h_1 + \rho_m g h - \rho_B g h_2 \quad (2.15)$$

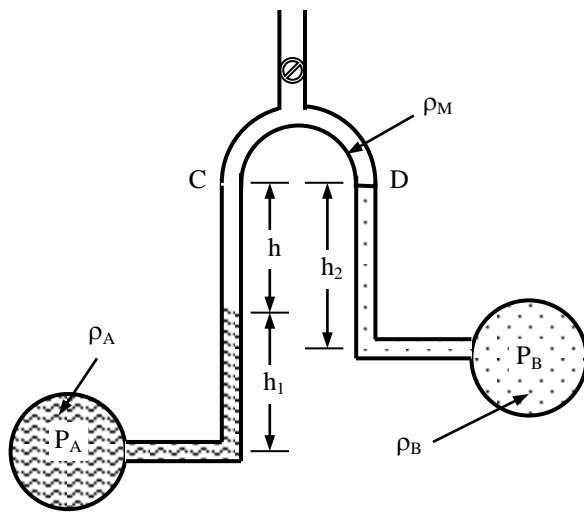


Fig. 2.8: Inverted U-Tube Manometer

If the top of the tube is filled with air, ρ_m will be negligible compared with ρ_A and ρ_B . Therefore Equation 2.15 becomes

$$P_A - P_B = \rho_A g h_1 - \rho_B g h_2 \quad (2.16)$$

If fluids in A and B are the same or they are two different point in a pipeline conveying the same fluid, therefore, $\rho_A = \rho_B = \rho$ and Equation 2.15 becomes

$$P_A - P_B = \rho g(h_1 - h_2) + \rho_m g h \quad (2.17)$$

If the top of the tube is filled with air and fluids in A and B are the same, therefore, Equation 2.15 becomes

$$P_A - P_B = \rho g(h_1 - h_2) \quad (2.18)$$

Example 2.6

An inverted U-tube of the form shown in Fig. 2.8 is used to measure the pressure difference between two points A and B in a pipeline through which water is flowing. The top of the manometer is filled with oil of relative density 0.8. Calculate the pressure difference, if h_1 is 0.25 m, h_2 is 0.45 m and h is 0.3 m.

Solution

The fluid in A and B is water therefore, $\rho = 1000\text{kg/m}^3$ and $\rho_m = 0.8 \times 1000 = 800\text{ kgm}^{-3}$

Pressure difference $P_A - P_B = \rho g(h_1 - h_2) + \rho_m g h$

$$\begin{aligned} P_A - P_B &= 1000 \times 9.81(0.25 - 0.45) + 800 \times 9.81 \times 0.3 \\ &= -1962 + 2354.4 = \mathbf{392.4\text{ Nm}^{-2}} \end{aligned}$$

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CHAPTER THREE

FORCES ON SUBMERGED SURFACES

In the design of dams, gates (hydraulic), tanks, ships etc. its important to compute the magnitude, direction and location of total forces acting upon the submerged surface. *A submerged surface can be defined as a surface of a body below the liquid surface.* There are two types of surfaces, namely:

- (i) Plane surface
- (ii) Curved surface

The former can be further classified as horizontal, vertical and inclined surface.

3.1 FORCES ON A SUBMERGED HORIZONTAL PLANE SURFACE

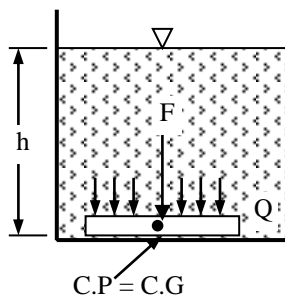


Fig. 3.1: Submerged horizontal plane surface

Consider a horizontal surface Q submerged in a liquid of density ρ at a depth h below the surface of the liquid. The pressure intensity at that level is

$$P = \rho gh \quad (3.1)$$

and the corresponding force acting on it is

$$F = \rho ghA \quad (3.2)$$

The total force F acts through a point on the plane called the centre of pressure.

3.2 FORCES ON A SUBMERGED VERTICAL PLANE SURFACE

Consider a surface of area A immersed vertically in a liquid of density, ρ . The pressure on one side of the surface is normal to the surface and given rise to a resultant force F on that side. Consider a thin elemental strip of the surface at depth h .

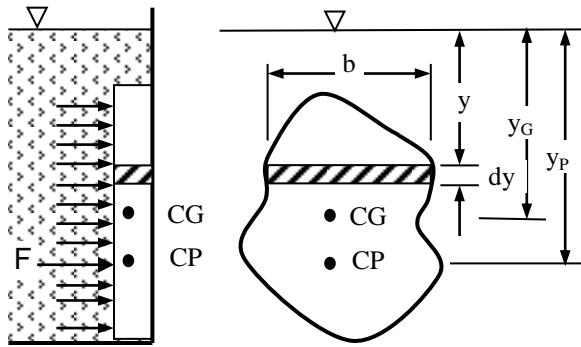


Fig. 3.2: Submerged vertical plane surface

y_G = centre of gravity of the area below the liquid surface

y = centre of gravity of elemental area

y_p = centre of pressure

The pressure at depth $y = \rho gy$

Area of the elemental strip under consideration

$$dA = bdy$$

Elemental force, $dF = PdA = \rho gy.bdy$

Total force on the plane surface is

$$\int dF = \int \rho gy.bdy = \rho g \int ydA$$

But $\int ydA$ is the first moment of area about the liquid surface

$$\text{Hence } F = \rho gAy_G \quad (3.3)$$

Determination of Centre of Pressure (y_p)

Let y_p be the depth of application of the force F . To determine y_p we shall take moment of elemental forces about the liquid surface, and the summation of these moments must be equal to Fy_p

$$dF = \rho gy.bdy = \rho gydA$$

Taking moment about the liquid surface

$$dF.y = \rho gy^2dA \quad \text{and} \quad \int dF.y = \rho g \int y^2dA$$

But the $\int y^2dA$ is the second moment of area I , about the surface level

$$Fy_p = \rho g \int y^2dA = \rho gI$$

$$y_p = \frac{\rho g I}{F} = \frac{\rho g I}{\rho g A y_G} = \frac{I}{A y_G}$$

$y_p = I/Ay_G =$ Second moment of Area/ First moment of Area

Using parallel axis theorem,

$$\begin{aligned} I_x &= I_G + A y_G^2 \\ I &= I_G + A y_G^2 \end{aligned} \quad (3.4)$$

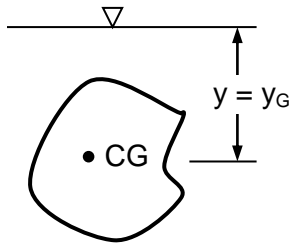


Fig. 3. 3: Central of gravity of submerged body

I_G is the second moment of Area about the centroid. Substituting for I , we have

$$y_p = \frac{I_G + A y_G^2}{A y_G}$$

$$\therefore y_p = \frac{I_G}{A y_G} + y_G \quad (3.5)$$

But $I_G = A K^2$ where $K =$ radius of gyration about axis GG (Fig. 3.4).

$$\therefore y_p = \frac{K^2}{y_G} + y_G \quad (3.6)$$

The geometric properties of some common shapes of planes are shown in Fig 3.4.

Example 3.1

A fuel tank 10 m wide by 5 m deep contains oil of relative density 0.7. In one vertical side a circular opening 1.8 m in diameter was made and closed by a trap door hinged at the lower end B held by a bolt at the upper end A. If the fuel level is 1.8 m above the top edge of the opening, calculate: (a) the total force on the door (b) the force on the bolt (c) the force on the hinge.

Solution (See Fig. 3.5)

$$\text{Relative density} = 0.7 \quad \rho_w = 1000 \text{ kgm}^{-3}$$

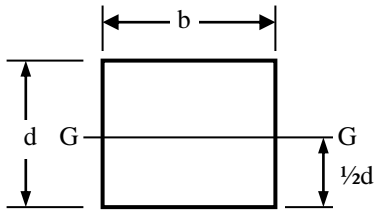
$$\rho_{oil} = RD \times \rho_w = 0.7 \times 1000 = 700 \text{ kgm}^{-3}$$

$$\text{Depth of centroid } y_G = \frac{1}{2}(1.8) + 1.8 = \mathbf{2.7 \text{ m}}$$

(a) Rectangle

$$A = bd$$

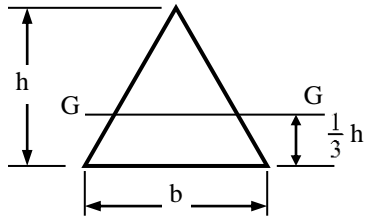
$$I_G = bd^3/12$$



(b) Triangle

$$A = \frac{1}{2} bh$$

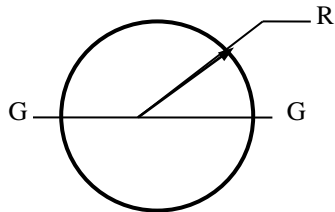
$$I_G = bh^3/36$$



(c) Circle

$$A = \pi R^2$$

$$I_G = \pi R^4/4$$



(d) Semicircle

$$A = \frac{1}{2} \pi R^2$$

$$I_G = 0.1102R^4$$

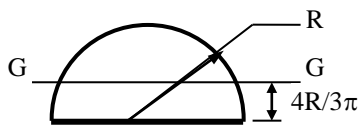


Fig 3.4: The geometric properties of some common shapes of planes.

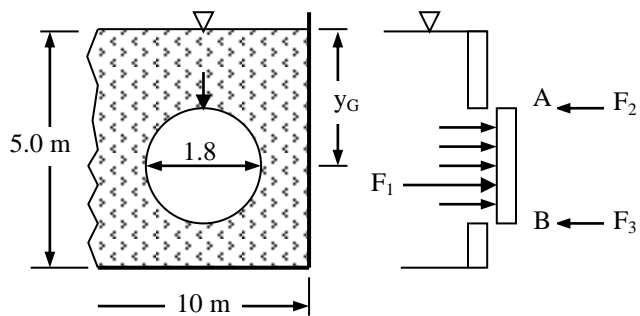


Fig. 3.5

Area of trap door, $\text{Area} = \pi d^2/4 = \pi(1.8)^2/4 = 2.54 \text{ m}^2$

(a) The total force on the door

$$F = \rho g A y_G$$

$$= 1000 \times 9.81 \times 2.545 \times 2.7 = \mathbf{47.2 \text{ kN}}$$

(b) The depth of application of force

$$y_p = I/Ay_G \text{ or } y_p = I_G/Ay_G + y_G$$

$$I = \pi R^4/4 = \pi(0.9)^4/4 = 0.5153$$

$$y_p = 0.5153/(2.545 \times 2.7) + 2.7$$

$$= 0.075 + 2.7 = \mathbf{2.775 \text{ m}}$$

Resolving all the forces horizontally

$$F_1 - F_2 - F_3 = 0$$

$$\therefore F_1 = F_2 + F_3 = \mathbf{47.2 \text{ kN}}$$

Taking moment of forces about A

$$F_3(3.6 - 1.8) - F_1(2.775 - 1.8) = 0$$

$$F_3 = \mathbf{25.57 \text{ kN}} \text{ (Force on the hinge)}$$

Hence, force on the bolt

$$F_2 = F_1 - F_3 = 47.2 - 25.57 = \mathbf{21.63 \text{ kN}}$$

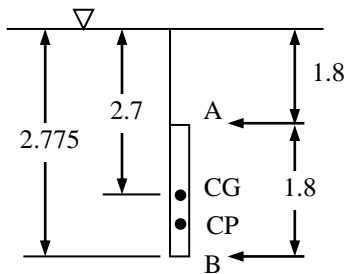


Fig.3.6

3.3 FORCES ON A SUBMERGED INCLINED PLANE SURFACE

The method of analysis of an inclined plane surface is similar to that discussed previously in Section 3.2 above. Consider a plane surface of an area A totally immersed in a liquid of density ρ . This surface is inclined at an angle θ to the horizontal and its centroid is at a vertical depth y_G below the free surface, the variable x is measured along the plane.

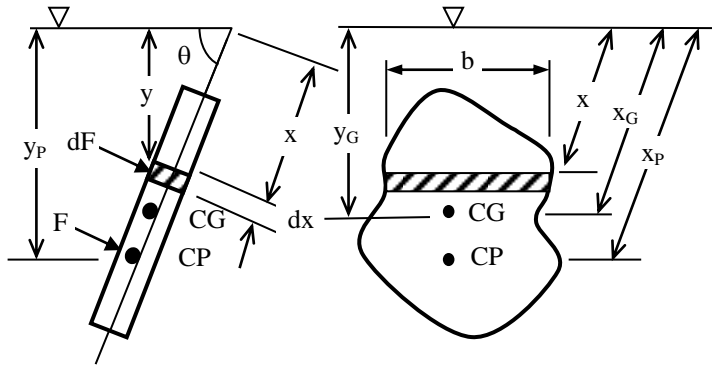


Fig.3.7: Submerged inclined plane surface

Let us consider the force on the element dx

$$dF = PdA \text{ and } P = \rho gy$$

but $y = x.\sin\theta$ and $dA = bdx$

therefore, $P = \rho gx.\sin\theta$

$$dF = \rho gx.\sin\theta.bdx \quad (3.7)$$

$$\int dF = \rho g.\sin\theta \int x.bdx = \rho g.\sin\theta \int x.dA$$

where $\int x.dA = Ax_G$ first moment of area.

$$F = \rho g \sin\theta Ax_G$$

$$\therefore F = \rho gy_G A \quad (3.8)$$

Where $y_G = x_G \sin\theta$

The centre of pressure can be determined as before

Taking moment about the fluid surface,

$$dM = x dF = \rho gx^2 \sin\theta b dx$$

$$\int dM = \rho g.\sin\theta \int x^2 b dx = \rho g.\sin\theta \int x^2 dA$$

$\int x^2 dA$ is the second moment of area about the free surface denoted by I ,

hence $M = \rho g.\sin\theta I$. The total moment can be expressed as:

$$M = Fx_P = \rho g.\sin\theta I.$$

$$x_P = \frac{\rho g \sin\theta I}{F}$$

But $F = \rho gy_G A = \rho gx_G.\sin\theta A$

$$x_P = \frac{\rho g \sin\theta I}{\rho gx_G \sin\theta A} = \frac{I}{Ax_G} \quad (3.9)$$

But $I = I_G + Ax_G^2$

$$\therefore x_P = \frac{I_G}{Ax_G} + x_G \quad (3.10)$$

Example 3.2

A triangular area 1.25 m by 2.0 m submerged in water is inclined at an angle 45° to the horizontal with its apex at 1.0 m below the surface of the water. Determine (a) the resultant force on the area and (b) the location of the force.

Solution

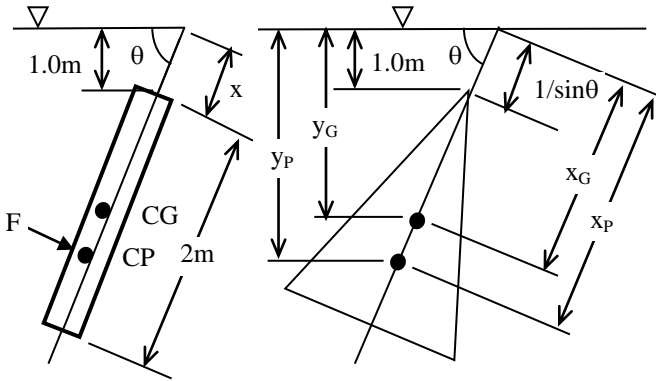


Fig. 3.8

$$x = 1/\sin\theta$$

Since CG is located at $\frac{1}{3}$ of height of triangle from the base, the upper portion is $\frac{2}{3}$ of the height.

$$x_G = \frac{2}{3} (2.0) + (\sin 45^\circ)^{-1} = 2.74 \text{ m}$$

$$\text{Area, } A = \frac{1}{2}bh = \frac{1}{2} \times 1.25 \times 2 = 1.25 \text{ m}^2$$

$$\begin{aligned} F &= \rho g \sin\theta \cdot A x_G \\ &= 9.81 \times 10^3 \sin 45^\circ \times 1.25 \times 2.747 \\ &= \mathbf{23819 \text{ N}} \end{aligned}$$

$$I_G = (1.25 \times 2^3)/36 = 0.2778 \text{ m}^4.$$

$$I = I_G + x_G^2 = 0.2778 + 1.25 \times 2.747^2 = \mathbf{9.71 \text{ m}^4}$$

$$\begin{aligned} x_P &= \frac{I}{A x_G} = \frac{9.71}{1.25 \times 2.747} \\ &= 2.828 \text{ m (measured along the inclined surface)} \end{aligned}$$

$$\begin{aligned} y_P &= x_P \cdot \sin\theta = 2.828 \sin 45^\circ \\ &= \mathbf{2.0 \text{ m}} \text{ (measured below the water level)} \end{aligned}$$

3.4 FORCES ON A SUBMERGED CURVED SURFACE

The normal pressure on a submerged curved surface varies in direction according to the curvature of the surface, and thus the previous method of analysis does not apply. The simplest method is to analyse the forces acting on the projected horizontal and vertical planes. These components may be subsequently combined into a resultant force, but it is unnecessary in many problems.

Consider the curve surface AB shown in Fig. 3.9. The section of fluid contained in ABDC is in equilibrium horizontally under the action of F_H and the resultant force F of the liquid on BN therefore, $F_H = F$. If the surface AB is removed and the space ABDE is filled with the liquid, this liquid would be in equilibrium under its own weight and the vertical force F_V on the boundary AB, therefore, Vertical component F_V is equal to weight of the fluid which would lie vertically above AB if AB were removed and F_V will act upwards through the centre of gravity G of the imaginary fluid.

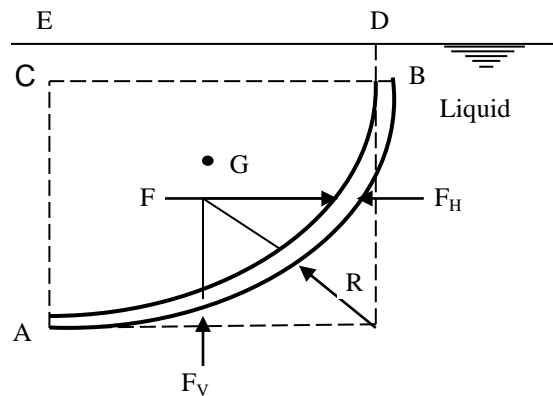


Fig. 3.9: Submerged curved surface

Example 3.3

A sluice gate (Fig. 3.10) consists of a quadrant of a circle of radius 1.5 m pivoted at its centre O . Its centre of gravity is at G as shown below. When the water is level with the pivot O , calculate the magnitude and direction of the resultant force on the gate due to water and the turning moment required to open the gate. The width of the gate is 3m and it has a mass of 6000 kg.

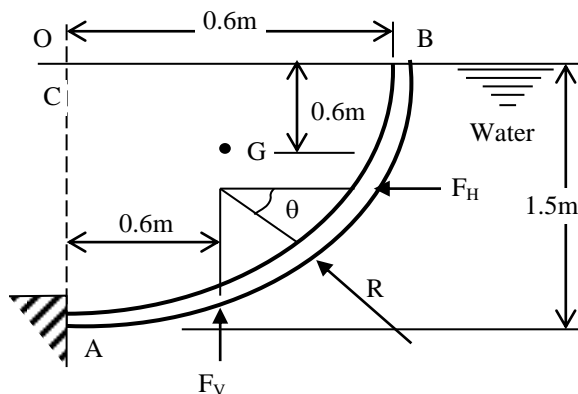


Fig. 3.10

Solution:

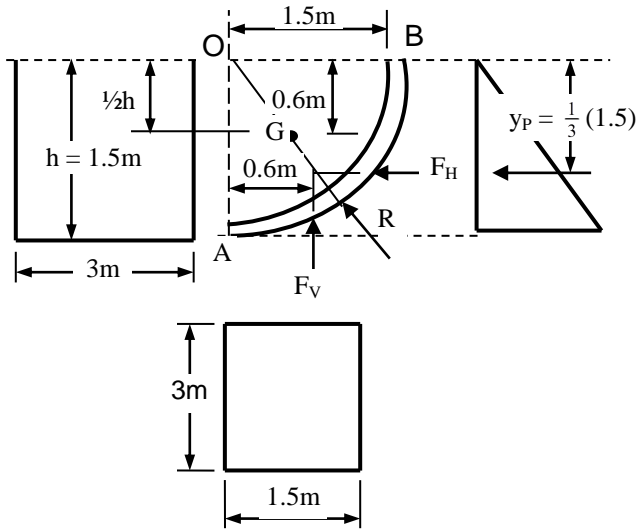


Fig. 3.11

Horizontal component F_H of resultant pressure is equal to Resultant pressure on projection OA of curved surface

$$F_H = \rho g \times \text{Area of OA} \times \text{depth to centroid of OA}$$

$$F_H = \rho g A (\frac{1}{2}h)$$

$$= 9.81 \times 10^3 \times (3 \times 1.5) \times 0.75 = \mathbf{33.1 \text{ kN}}$$

Vertical component F_V is equal to the weight of fluid which would occupy OAB

$$F_V = \rho g \times \text{volume of cylindrical sector OAB} = \rho g \cdot V$$

But Area of quadrant, $A = \frac{1}{4}\pi r^2$ and

$$\text{Volume} = \text{Area} \times \text{width} = \frac{1}{4}\pi r^2 L$$

$$\therefore F_V = \rho g (\frac{1}{4}\pi r^2 L)$$

$$= 9.81 \times 10^3 \times \frac{1}{4} \pi (1.5)^2 \times 3$$

$$= \mathbf{52007 \text{ N or } 52 \text{ kN}}$$

$$\text{Resultant force} = (F_V^2 + F_H^2)^{1/2} \quad (3.11)$$

$$R = [(52)^2 + (33.1)^2]^{1/2} = \mathbf{61.6 \text{ kN}}$$

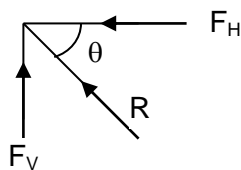


Fig. 3.12

If θ is the angle of inclination of R to the horizontal i.e.

$$\tan\theta = F_V/F_H = 52/33.1$$

$$\theta = 57.5^\circ$$

Since the surface is part of a cylinder, R will act through O. Therefore R has no moment about the part O.

The only turning moment is that due to the weight of the gate.

Turning moment required to open gate = $W \times 0.6$

$$= 6000 \times 9.81 \times 0.6 = \mathbf{35.3 \text{ kNm.}}$$

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CHAPTER FOUR

BUOYANCY AND STABILITY OF FLOATING BODIES

The method of calculating the forces on a curved surface applies to all shapes of surface and, therefore, to the surface of a totally submerged object. But most problems concerned with totally or partially immersed bodies are problems of equilibrium between forces due to the weight of the body. However when a body is floating, such as a ship, it is not sufficient that the body should be in equilibrium, it must also be in stable equilibrium (in the desired position) so that when it rolls or pitches a restoring moment will be produced which will tend to return the body to its equilibrium position.

4.1 BUOYANCY

In accordance with principle of Archimedes for buoyancy and floatation which states that the upthrust (upward vertical force due to the fluid) or buoyancy of an immersed body is equal to the weight of liquid displaced. A submerged body will rise to the surface provided that the weight of the body is less than the weight of the displaced liquid, otherwise it will sink. The centre of gravity of the displaced liquid is called the *centre of buoyancy*.

4.2 EQUILIBRIUM OF FLOATING BODIES,

When a body floats in vertical equilibrium in a liquid, the forces present are the upthrust R acting through the centre of buoyancy B and the weight of the body $W = mg$ acting through its centre of gravity. For equilibrium, R and W must be equal to the weight of fluid displaced, ρgV , where V is the volume of fluid displaced; therefore,

$$R = W \text{ i.e. } \rho gV = mg \text{ and } V = mg/\rho g = m/\rho$$

$$\text{Volume of fluid displaced} = \frac{\text{mass of the floating body}}{\text{density of the fluid}} \quad (4.1)$$

4.3 STABILITY OF A SUBMERGED BODY.

For stability of a submerged body the centre of gravity of the body must lie directly below the centre of buoyancy of the displaced liquid. Should the two points coincide, the submerged body is in neutral equilibrium for all positions.

4.4 STABILITY OF FLOATING BODIES

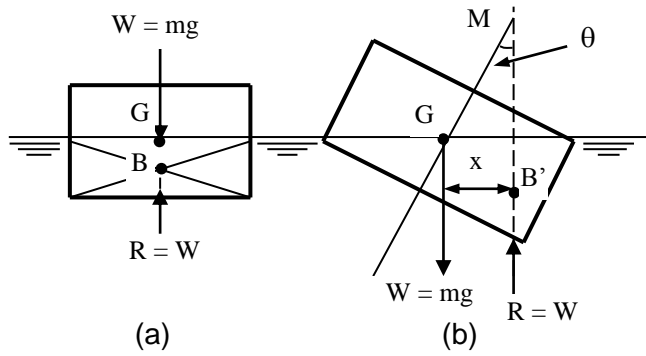


Fig. 4.1: A body floating in a stable equilibrium

The above figure shows a body floating in equilibrium. The weight $W = mg$ acts through the centre of gravity G and the upthrust R acts through the centre of buoyancy B of the displaced fluid in the same straight line as W . When the body is displaced through an angle θ (Fig. 4.1), W continues to act through G the volume of liquid remains unchanged since $R = W$, but the shape of this volume change and its centre of gravity, which is the centre of buoyancy moves relative to the body from B to B' .

Since R and W are no longer in the same straight line, a turning point proportional to $W \times \theta$ is produced, which in Fig (b) is righting moment. If M is the point at which the line of action of the upthrust R cuts the original vertical through the centre of gravity G , $x = GM \cdot \sin\theta$

But if the angle of tilt θ is small, so that

$$\sin\theta = \tan\theta = \theta,$$

$$\therefore x = GM \cdot \theta$$

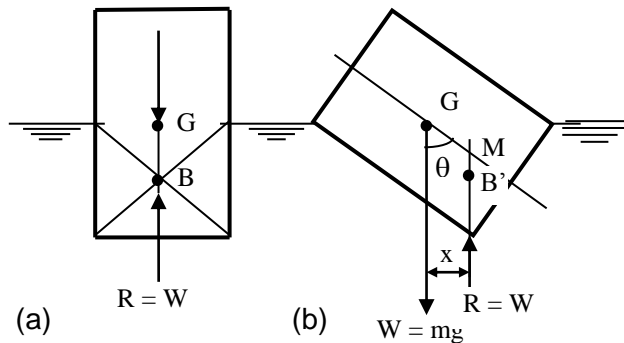


Fig. 4.2: A body floating in an unstable equilibrium

The point M is called the metacentre and the distance GM is the metacentric height.

From Fig. 4.1 and Fig.4.2 (b), it can be seen that

- (i) if M lies above G , a righting moment $W \cdot GM \cdot \theta$ is produced, equilibrium is stable and GM is regarded as positive.

- (ii) If M lies below G, an overturning moment $W.GM.\theta$ is regarded as negative.
- (iii) If M coincides with G, the body is in neutral equilibrium.

Since a floating body can tilt in any direction, it is usual, for a ship, to consider displacement about the longitudinal (rolling) and transverse (pitching) axes. The position of the metacentre and the value of the metacentric height will normally be different for rolling and pitching.

4.5 DETERMINATION OF POSITION OF METACENTRE RELATIVE TO THE CENTRE OF BUOYANCY.

Metacentre: The metacentre is the point at which the line of action of upthrust (or buoyant force) for the displaced position intercept the original Vertical axis through the centre of gravity of the body.

Metacentric Height: The distance of metacentre from the centre of gravity of the body is called metacentric height.

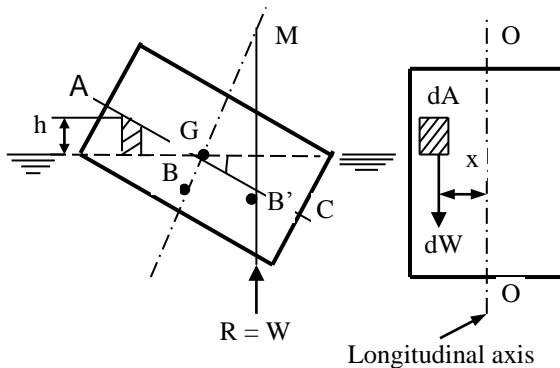


Fig. 4.3: Metacentre and central of bouoyancy of a floating body

For a vessel of known shape and displacement, the position of the centre of buoyancy B is comparatively easily found and the position of the mental centre M relative to B can be calculated as follows. AC is the original waterline plane and B the centre of buoyancy in the equilibrium position. When the vessel is tilted through a small angle θ , the centre of buoyancy will move to B' as a result of the alteration in the shape of the displaced fluid.

Consider an elemental horizontal area dA of height 'h' (Fig. 4.3),

$$h = x.\tan\theta$$

The weight of the elemental volume of fluid

$$dW = \rho gh.dA = \rho gx \tan\theta.dA$$

Taking moment about the longitudinal axis OO

$$dM = x.dW = \rho g.x^2\tan\theta.dA$$

$$\text{Total moment, } M = \int dM = \rho g\tan\theta \int x^2dA$$

Where $\int x^2dA = I =$ second moment of area of waterline plane about OO.

$$\text{Therefore, } M = \rho g\tan\theta.I \tag{4.2}$$

The buoyance moment (Fig. 4.4) = $.R.BB'$

but $BB' = BM.\sin\theta$

If V is the volume of the fluid displaced then

Buoyant force $R = \rho gV$.

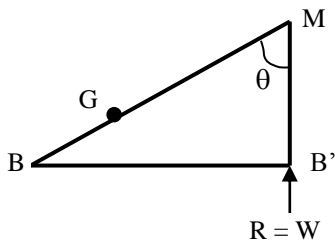


Fig. 4.4

Hence, Buoyant moment = RBB'

$$M_B = \rho gV.BM\sin\theta \quad (4.3)$$

Equating Eqs. 4.2 and 4.3, we have $\rho g\tan\theta.I = \rho gV.BM\sin\theta$

$$BM = \frac{I \tan\theta}{V \sin\theta}$$

If the angle of tilt is very small, $\sin\theta = \tan\theta$

$$\text{therefore, } BM = \frac{I}{V} \quad (4.4)$$

The distance BM is known as the metacentric radius

$$\text{But } GM = BM - BG = (I/V) - BG \quad (4.5)$$

Example 4.1

A stone weighs 400 N in air, and when immersed in water it weighs 222 N. Compute the volume of the stone and its relative density.

Solution

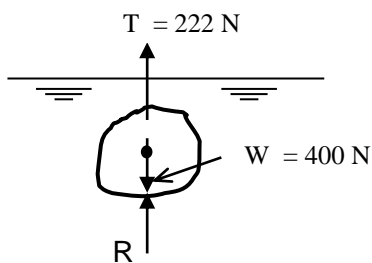


Fig. 4.5

Resolving forces vertically

$$T + R = W \text{ or } R = W - T$$

$$R = 400 - 222 = 178 \text{ N}$$

Volume of the stone is equal to the volume of water displaced by the stone. But the weight of liquid displaced is given by $R = \rho g V$

$$V = \frac{R}{\rho g} = \frac{178}{1000 \times 9.81}$$

$$V = \mathbf{0.018 \text{ m}^3} \text{ (volume of water displaced by the stone)}$$

Relative density:

$$RD = \frac{\text{weight of substance}}{\text{weight of equal volume of water}}$$

$$RD = \frac{400}{178} = \mathbf{2.25}$$

Example 4.2

A pontoon is 6m long, 3m wide 3m deep, and the total weight is 260 kN. Find the position of the metacentre for rolling in sea water. How high may the centre of gravity be raised so that the pontoon is in neutral equilibrium? (Take density of sea water to be 1025 kgm^{-3})

Solution

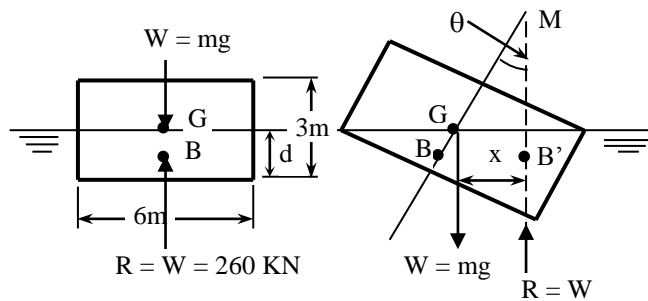


Fig. 4.6

The total volume of water displaced $V = R/\rho g$

$$V = \frac{R}{\rho g} = \frac{260 \times 10^3}{1025 \times 9.81} = 25.86 \text{ m}^3$$

The depth of displacement,

$$d = \frac{V}{A} = \frac{25.86}{6 \times 3} = 1.44 \text{ m}$$

$$BM = \frac{I}{V}$$

$$\text{but } I = \frac{bh^3}{12} = \frac{6 \times 3^3}{12} = 13.5 \text{ m}^4$$

$$BM = \frac{135}{25.86} = 0.522 \text{ m}$$

Height $M = BM + \frac{1}{2}d$

$$M = 0.522 + \frac{1}{2}(1.44) = \mathbf{1.242 \text{ m}}$$

For the pontoon to be in neutral equilibrium position, M must coincide with G , that is $M = G$. The centre of G must be located at distance 1.242m above the base

$$B = \frac{1}{2}d = \frac{1}{2} \times 1.44 = 0.72 \text{ m}$$

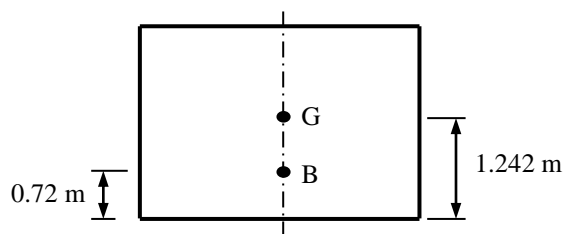


Fig. 4.7

Example 4.3

A cylindrical buoy 1.35 m in diameter and 1.8 m high has a mass of 770 kg. Show that it will not float with its axis vertical in the sea water of density 1025 kgm^{-3} . If one end of a vertical chain is fastened to the base, find the pull required to keep the buoy vertical. The centre of gravity of the buoy is 0.9 m from its base.

Solution

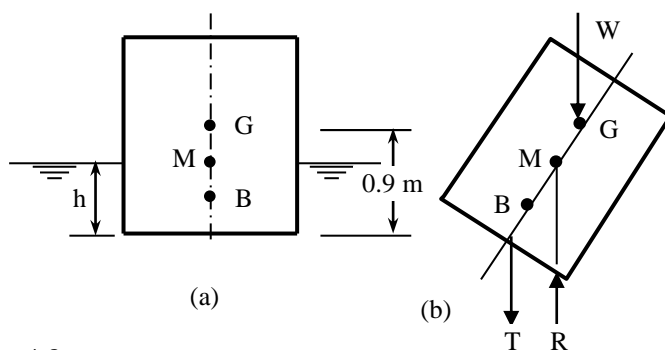


Fig. 4.8

(a) Without the anchor chain (Fig. 4.8a)

Mass of the buoy is equal to the mass of the water displaced.

Volume of water displaced,

$$V = \frac{m}{\rho} = \frac{770}{1025} = 0.75 \text{ m}^3$$

But the volume, $V = \pi r^2 h$

Therefore, depth of immersed,

$$h = \frac{V}{\pi r^2} = \frac{0.75}{\pi(0.675)^2} = 0.524 \text{ m}$$

Height of centre of buoyancy above base = $\frac{1}{2}h = 0.262 \text{ m}$

Height of centre of gravity above base = 0.9 m

Therefore, $BG = 0.9 - 0.262 = 0.638 \text{ m}$

Also, $BM = I/V$.

The second moment of area for a circular body

$$I = \frac{1}{4}\pi r^4 = \frac{1}{4}\pi(0.675)^4 = 0.1630 \text{ m}^4$$

$$BM = \frac{I}{V} = \frac{0.1630}{0.75} = 0.2174 \text{ m}$$

The metacentric height, $GM = BM - BG$

$$GM = 0.2174 - 0.638 = -0.42 \text{ m}$$

The negative metacentric height indicates that the buoy is unstable; hence, it will not float with its axis vertical.

(b) *With the anchor chain (Fig. 4.8b)*

If T = pull in the chain in Newtons,

New upthrust, $R = T + W$

New displacement volume,

$$V = \frac{R}{\rho g} = \frac{R}{1025 \times 9.81} = \frac{R}{10055.25} \text{ m}^3$$

New draught, $h = R/\rho g A$

$$h = \frac{R}{\rho g A} = \frac{R}{1025 \times 9.81 \times \pi(0.675)^2} = \frac{R}{14400} \text{ m}$$

Height of centre of buoyancy above $A = \frac{1}{2}h$

$$\frac{1}{2}h = R/28800 \text{ m}$$

$$BM = \frac{I}{V} = \frac{0.163}{R/10055.25} = \frac{1639}{R}$$

$$AM = AB + BM = \frac{R}{28800} + \frac{1639}{R} \text{ m}$$

$$AG = 0.9 \text{ m}$$

$$GM = AG - AM = 0.9 - \left(\frac{R}{28800} + \frac{1639}{R} \right)$$

For equilibrium, taking moments about G,

$$0.9T = R \left[0.9 - \left(\frac{R}{28800} + \frac{1639}{R} \right) \right]$$

But $R = T + W$ where $W = mg$

Tension in the chain, $T = R - mg$

or $T = R - 770 \times 9.81 = R - 7553.7$

$$\therefore 0.9(R - 7553.7) = 0.9R - \frac{R^2}{28800} - 1639$$

$$R^2 = 28800(6798 - 1639) = 149 \times 10^6$$

$$R = \mathbf{12200 \text{ N}}$$

$$T = 12200 - 770 \times 9.81 = \mathbf{4640 \text{ N}}$$

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CHAPTER FIVE

FLUID FLOW AND EQUATION

There is no shear force in a fluid at rest but, when it is in motion, shear forces can be set up due to viscosity and turbulence, which oppose motion, producing a “frictional” effect. Many problems can be solved, at least partially, by assuming an ideal frictionless (inviscid) fluid.

5.1 DYNAMICS OF FLUID FLOW

The study of fluid dynamics deals with the forces responsible for fluid motion and the resulting accelerations. A fluid in motion experiences in addition to gravity, pressure forces, viscous and turbulent shear resistances, boundary resistance, and forces due to surface tension and compressibility effects of the fluid. The presence of such a complex system of forces in real fluid flow problems makes the analysis very complicated.

However, a simplifying approach to the problem may be made by assuming the fluid to be ideal or perfect, i.e. non-viscous or frictionless and incompressible. Water has a relatively low viscosity, is practically incompressible, and is found to behave like an ideal fluid.

When fluid is flowing along a pipe the motion is normally turbulent and individual particle does not follow a regular path. The velocity is approximately uniform across the section except very close to the pipe wall and it is usual to assume a uniform velocity across the whole section.

Boundary Layer: The layer of fluid in the immediate neighbourhood of an actual flow boundary that has had its velocity relative to the boundary affected by viscous shear is called the *boundary layer*. Boundary layers may be laminar or turbulent, depending generally upon their length, the viscosity, the velocity of the flow near them and the boundary roughness.

Adiabatic Flow: Adiabatic flow is that flow of a fluid in which no heat is transferred to or from the fluid. *Reversible adiabatic* (frictionless adiabatic) flow is called isentropic flow.

Streamline: A streamline is a continuous line drawn through the fluid so that it has the direction of the velocity vector at every point (Fig. 5.1).

Stream Tube: A stream tube is the tube made by all the streamlines passing through a small, closed curve (Fig. 5.2).

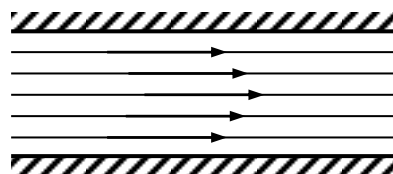


Fig. 5.1: Streamline

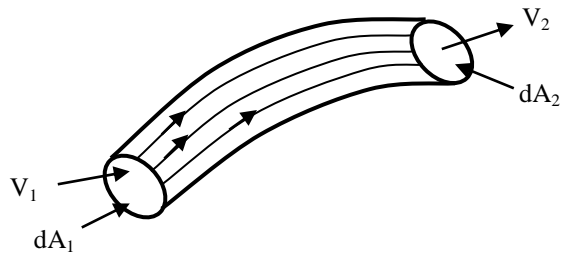


Fig. 5.2: Stream tube

5.2 DISCHARGE AND MEAN VELOCITY

The total quantity of fluid flowing in a unit time past any particular cross-section of a stream is called the '*discharge*' or flow at that section. It can be measured either in terms of mass, in which case it is referred to as the *mass flow rate* (m) or in terms of volume, when it is referred to as the volumetric flow rate or discharge (Q).

Volumetric Flow rate or Discharge (Q): It is defined as the volume of fluid passing a given cross-section in unit time. It is measured in cubic metres per second, (m^3s^{-1}).

Mass Flow Rate (m): It is defined as the mass of fluid passing a given cross-section in unit time. It is measured in kilograms per second (kgs^{-1}).

Mean Velocity: The mean velocity at any cross-section area is the ratio of volumetric flow rate to the cross-sectional area, i.e. $V = Q/A$.

5.3 THE CONCEPTS OF SYSTEM AND CONTROL VOLUME

System: A system refers to a definite mass of material and distinguishes it from all other matter called surroundings. Therefore, *a system can be defined as a collection of matter which is separated from the surrounding matter by a boundary and can interact with the surrounding matter via the boundary*. The boundaries of a system form a closed surface. This surface may vary with time, so that it contains the same mass during changes in its condition. The system may contain an infinitesimal mass or a large finite mass and solids at the will of the investigator.

The Law of Conservation of Mass

The law of conservation of mass states that the mass within a system remains constant with time disregarding relativity effects. Therefore,

$$\frac{dm}{dt} = 0 \quad (5.1)$$

where m is the total mass.

Conservation of Momentum

The conservation of (linear) momentum is expressed through Newton's second law of motion as:

$$\sum F = \frac{d}{dt}(mv) \quad (5.2)$$

where m is the constant mass of the system. $\sum F$ refers to the resultant of all external forces acting on the system, including body forces such as gravity, and 'v' is the velocity of the centre of mass of the system.

Control Volume

A control volume refers to a region in space and is useful in the analysis of situations where flow occurs into and out of the space. The boundary of a control volume is its control surface. The content of the control volume is called the system.

5.4 CONTINUITY EQUATION

The continuity equation is developed from the general principle of conservation of mass, Equation (5.1). *The continuity equation for a control volume states that the time rate of increase of mass within a control volume is just equal to the net rate of mass inflow to the control volume.*

Consider the flow through an infinitesimal stream tube represented in Fig. 5.2. Since the cross sectional areas at inlet and outlet are very small, it can be assumed that the flow velocity is uniform over each cross section. The density and the velocity are ρ_1 and v_1 for the inlet section and ρ_2 and v_2 for the outlet section. Consequently, the corresponding mass flow rates are $\rho_1 v_1 dA_1$ and $\rho_2 v_2 dA_2$ respectively. For conservation of mass, the rate of increase of mass in the system must be equal to the net inflow of mass into the control volume. Therefore,

$$\frac{dm}{dt} = \rho_1 v_1 dA_1 - \rho_2 v_2 dA_2 \quad (5.3)$$

where dm/dt is the rate of increase of mass of the system due to net inflow. Since a system is one whose mass is always constant with respect to time, therefore, dm/dt is equal to zero, Equation (5.3) reduces to:

$$\rho_1 v_1 dA_1 = \rho_2 v_2 dA_2 \quad (5.4)$$

Equation (5.4) can be extended to large cross sections by integrating the mass flow rate across each cross section:

$$\int_{A_1} \rho_1 v_1 dA_1 = \int_{A_2} \rho_2 v_2 dA_2 \quad (5.5)$$

If ρ and v are constant over each section, the equation reduces to:

$$\rho_1 v_1 A_1 = \rho_2 v_2 A_2 \quad (5.6)$$

For an incompressible fluid, the density is constant, ($\rho_1 = \rho_2$) and Equation (5.6) becomes

$$v_1 A_1 = v_2 A_2 = Q \quad (5.7)$$

Therefore, the volumetric flow rate or discharge (Q) is constant from section to section as long as the same flow goes through all the sections. Equation (5.7) shows that the flow velocity (v) is inversely proportional to the flow area (A).

The control volume concept can be applied to continuity equation (Equation 5.7). For a control volume with many outlets and inlets ports, the sum of the flows through the inlet must be equal to the sum of the flows through the outlet ports. Therefore,

$$\sum_{a=1}^p Q_a = \sum_{b=1}^q Q_b \quad (5.8)$$

where p is the number of inlet ports; and q is the number of outlet ports.

Example 5.1

Water flows through a pipe line of 700 mm diameter at A and then branches into two pipes, one branch being 350 mm diameter discharging B and the other branch 280 mm diameter discharging at C. If the velocity at A is 2 ms^{-1} and the velocity at B is 4.5 ms^{-1} , determine the discharge at A, B and C, and the velocity at C.

Solution

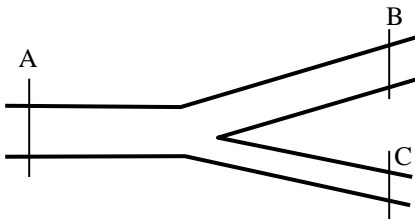


Fig. 5.3

$$Q_A = Q_B + Q_C$$

Discharge:

$$\text{at A, } Q_A = V_A A_A = 2 \times \frac{1}{4}\pi(0.7)^2 = \mathbf{0.77 \text{ m}^3\text{s}^{-1}}$$

$$\text{at B, } Q_B = V_B A_B = 4.5 \times \frac{1}{4}\pi(0.35)^2 = \mathbf{0.433 \text{ m}^3\text{s}^{-1}}$$

$$\text{at C, } Q_C = Q_A - Q_B = 0.77 - 0.433 = \mathbf{0.337 \text{ m}^3\text{s}^{-1}}$$

$$\therefore V_C = Q_C / A_C = 0.337 / \frac{1}{4}\pi(0.28)^2 = \mathbf{5.47 \text{ ms}^{-1}}$$

5.5 ENERGY EQUATION FOR AN IDEAL FLUID FLOW

Consider an elemental stream tube in motion along a streamline (Fig. 5.4) of an ideal fluid flow. The forces responsible for its motion are the pressure forces, gravity and accelerating force, due to change in

velocity along the streamline. All frictional forces are assumed to be zero and the flow is irrotational i.e. Uniform velocity distribution across streamlines.

Since Force = Mass x Acceleration or Pressure x Area

$$\text{Therefore, } PdA - (P + dP)dA - \rho g dA ds \cos \theta = \rho dA ds \left(\frac{dv}{dt} \right)$$

$$\text{or } -dP - \rho g ds \cos \theta = \rho ds \left(\frac{dv}{dt} \right)$$

The tangential acceleration (along streamline) for steady flow,

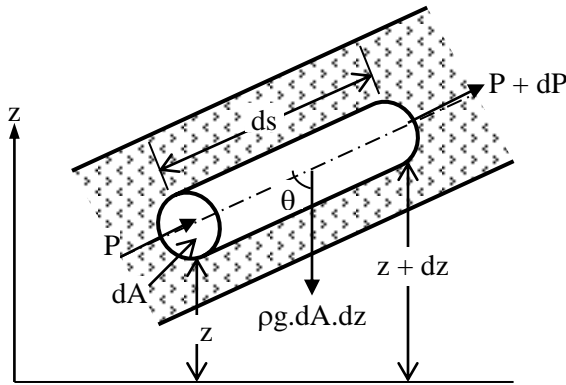


Fig. 5.4: Euler's equation of motion.

$$\frac{dv}{dt} = v \left(\frac{dv}{ds} \right) \quad \text{since } \frac{dv}{dt} = \frac{ds}{dt} \times \frac{dv}{ds}$$

$$\text{and } \cos \theta = \frac{dz}{ds}$$

$$\text{Therefore, } -dP - \rho g dz = \rho v dv$$

divide through by ρg to obtain Equation (5.9):

$$dz + \frac{dP}{\rho g} + \frac{dv^2}{2g} = 0 \quad \text{since } (dv^2 = 2v dv)$$

$$\text{or } dz + \frac{dP}{\rho g} + \frac{d(v)^2}{2g} = 0 \quad (5.9)$$

Equation (5.9) is the **Euler equation** of motion applicable to steady state, irrotational flow of an ideal incompressible fluid.

On integration along the streamline, we get:

$$z + \frac{P}{\rho g} + \frac{v^2}{2g} = \text{constant} \quad (5.10)$$

$$\text{or} \quad z_1 + \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = z_2 + \frac{P_2}{\rho g} + \frac{v_2^2}{2g} \quad (5.11)$$

where z = elevation; P = Pressure; and v = average (uniform) velocity of the fluid at a point in the flow under consideration. Equation (5.11), known as *Bernoulli's equation*, is sometimes called the energy equation for steady ideal fluid flow along a streamline between two sections 1 and 2.

Bernoulli's theorem states that the total energy of all points along a steady continuous stream line of an ideal incompressible fluid flow is constant although its division between the three forms of energy may vary and it is written as Equation (5.10). The three terms on the left-hand side of Equation 5.10 have the dimension of length or head and the sum can be interpreted as the total energy of a fluid element of unit weight.

The first term z , is referred to as the *potential head* of the liquid. The second term $P/\rho g$, is referred to as the *pressure head* and the third term $v^2/2g$, is referred to as the *velocity head*. The addition of the three heads is constant and it is referred to as total head H .

Total head = potential head + Pressure head + Velocity head

$$H = z + \frac{P}{\rho g} + \frac{v^2}{2g} \quad (5.12)$$

where H is the total energy per unit weight.

Potential Head (z): Potential head is the potential energy per unit weight of fluid with respect to an arbitrary datum of the fluid. z is in JN^{-1} or m

Pressure Head ($P/\rho g$): Pressure head is the pressure energy per unit weight of fluid. It represents the work done in pushing a body of fluid by fluid pressure.

$P/\rho g$ is in JN^{-1} or m.

Velocity Head ($v^2/2g$): Velocity head is the kinetic energy per unit weight of fluid in JN^{-1} or m.

In formulating Bernoulli's equation (Equation 5.11), it has been assumed that no energy has been supplied to or taken from the fluid between points 1 and 2. Energy could have been supplied by introducing a pump; equally, energy could have been lost by doing work against friction or in a machine such as a turbine. Bernoulli's equation can be expanded to include these conditions, giving

$$z_1 + \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = z_2 + \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h + w - q \quad (5.13)$$

where h is the loss per unit weight; w is the work done per unit weight; q is the energy supplied per unit weight

5.6 THE POWER OF A STREAM OF FLUID

The total energy per unit weight H of the fluid is given by (Equation 5.12). If the volume rate of flow (Q) is known and the density of the fluid is ρ , therefore weight per unit time of fluid flowing can be calculated using Equation (5.14).

$$\text{Weight per unit time} = \rho g Q \text{ (Ns}^{-1}\text{)} \quad (5.14)$$

Therefore, power of fluid flowing can be calculated as the product of energy per unit weight H (in m or JN^{-1}) and weight per unit time in N s^{-1} .

Power = Energy per unit time

$$= (\text{weight/unit time}) \times (\text{energy/unit weight})$$

$$\text{Power} = \rho g Q H \quad (\text{W or kW}) \quad (5.15)$$

5.7 SEPARATION AND CAVITATION IN FLUID FLOW

5.7.1 Separation in Fluid Flow

Consider a rising main (Fig. 5.5) of uniform pipeline. For a given discharge in Fig. 5.5, the velocity is the same at all sections (uniform diameter) and hence $z + P/\rho g = \text{constant}$. As the elevation ' z ' increases, the pressure ' P ' in the system decreases and if pressure becomes vapour pressure of the fluid, the fluid tends to boil liberating dissolved gases and air bubbles. With further liberation of gases the bubbles tend to grow in size eventually blocking the pipe section thus allowing the discharge to take place intermittently. This phenomenon is known as *separation* and greatly reduces the efficiency of the system.

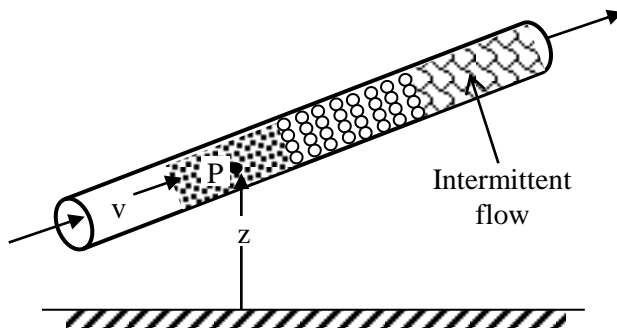


Fig. 5.5: Rising main of uniform diameter.

$$\text{Total energy} = z + \frac{P}{\rho g} + \frac{v^2}{2g} = \text{constant} \quad (5.16)$$

5.7.2 Cavitation in Fluid Flow

If the tiny air bubbles in Fig. 5.6 formed at the separation point are carried to a high pressure region by the flowing fluid, they collapse extremely abruptly or implode producing a violent hammering action

on any boundary surface on which the imploding bubbles come in contact and cause pitting and vibration to the system which is highly undesirable. The whole phenomenon is called *cavitation* and should be avoided while designing any hydraulic system.

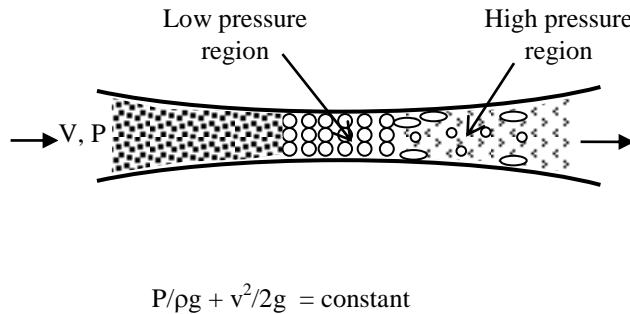


Fig. 5.6: Horizontal converging – diverging pipe.

Example 5.2

Water is flowing along a pipe with a velocity of 7.2 ms^{-1} . Express this as a velocity head in metres of water. What is the corresponding pressure in kNm^{-2} ?

Solution

$$\begin{aligned} \text{(i) Velocity head} &= v^2/2g = (7.2)^2/(2 \times 9.81) \\ &= 51.84/19.62 = \mathbf{2.64 \text{ m}} \end{aligned}$$

$$\text{(ii) Pressure in } \text{kN/m}^2, v^2/2g \times \rho g = \frac{1}{2}\rho v^2$$

$$\frac{1}{2}\rho v^2 = \frac{1}{2}(1000 \times 51.84) = 25920 \text{ Nm}^{-2} = \mathbf{25.9 \text{ kNm}^{-2}}$$

Example 5.3

Water in a pipeline 36m above sea level is under a pressure of 410 kN/m^2 and the velocity of flow is 4.8 m/s. Calculate the total energy per unit weight reckoned above sea level.

Solution

$$H = z + P/\rho g + v^2/2g$$

$$\begin{aligned} H &= 36 + (410 \times 10^3)/(1000 \times 9.81) + (4.8)^2/2 \times 9.81 \\ &= 36 + 41.79 + 1.17 = \mathbf{78.96 \text{ m or } 78.96 \text{ JN}^{-1}} \end{aligned}$$

Example 5.4

Water flow from a reservoir into a closed tank in which the pressure is 70 kNm^{-2} below atmospheric. If the water level in the reservoir is 6 m above that in the tank, find the velocity of water entering the tank, neglecting friction.

Solution

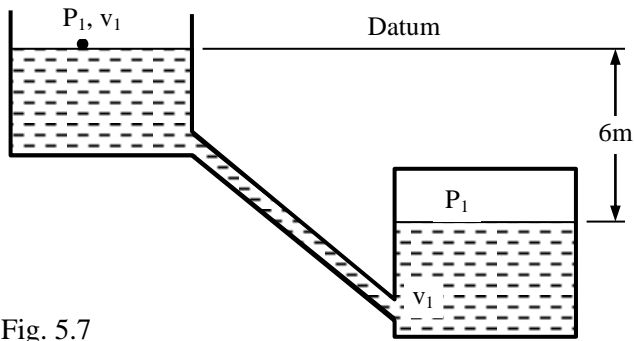


Fig. 5.7

$$z_1 + \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = z_2 + \frac{P_2}{\rho g} + \frac{v_2^2}{2g}$$

$$z_1 = 0, v_1 = 0, P_1 = P_{\text{atm}}, z_2 = -6 \text{ m and } P_{\text{atm}} - P_2 = 70 \text{ kNm}^{-2}$$

$$\therefore 0 + P_{\text{atm}}/\rho g + 0 = -6 + P_2/\rho g + v_2^2/2g$$

$$P_{\text{atm}}/\rho g - P_2/\rho g + 6 = v_2^2/2g$$

$$(P_{\text{atm}} - P_2)/\rho g + 6 = v_2^2/2g$$

$$(70 \times 10^3)/(10^3 \times 9.81) + 6 = v_2^2/(2 \times 9.81)$$

$$v_2^2 = (70 \times 2) + (6 \times 2 \times 9.81) = 140 + 117.72 = 257.72$$

$$\therefore v_2 = (257.72)^{1/2} = \mathbf{16.05 \text{ ms}^{-1}}$$

Example 5.5

A siphon has a uniform circular bore of 75 mm diameter and consists of a bent pipe with its crest 1.8 m above water level discharging into the atmosphere at a level 3.6 m below water level. Find the velocity of flow, the discharge and the absolute pressure at crest level if the atmospheric pressure is equivalent to 10 m of water. Neglect losses due to friction.

Solution

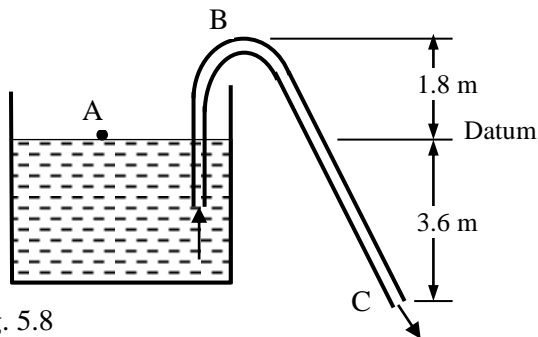


Fig. 5.8

$$(i) z_A + \frac{P_A}{\rho g} + \frac{v_A^2}{2g} = z_C + \frac{P_C}{\rho g} + \frac{v_C^2}{2g} \quad \text{or}$$

$$0 + \frac{P_{atm}}{\rho g} + 0 = -3.6 + \frac{P_{atm}}{\rho g} + \frac{v_C^2}{2g}$$

$$\therefore v_C^2 = 3.6 \times 2g = 3.6 \times 2 \times 9.81 = 70.632$$

$$v_C = (70.632)^{1/2} = \mathbf{8.4 \text{ ms}^{-1}}$$

$$(ii) Q = AV = \frac{1}{4}\pi d^2 v$$

$$Q = \frac{1}{4}\pi(0.075)^2 \times 8.4 = \mathbf{0.0371 \text{ m}^3\text{s}^{-1}}$$

$$(iii) z_A + \frac{P_A}{\rho g} + \frac{v_A^2}{2g} = z_B + \frac{P_B}{\rho g} + \frac{v_B^2}{2g} \quad \text{or}$$

$$0 + 10 + 0 = 1.8 + \frac{P_B}{\rho g} + \frac{(8.4)^2}{2g}$$

$$P_B/\rho g = 10 - 1.8 - (8.4)^2/(2 \times 9.81) = 4.6$$

$$P_B = 4.6 \times 1000 \times 9.81$$

$$P_B = 45126 \text{ Nm}^{-2} \quad \text{or} \quad \mathbf{45.1 \text{ kNm}^{-2}}$$

Example 5.6

A pipe carrying water tapers from 160 mm diameter at A to 80 mm diameter at B. Point A is 3 m above B. The pressure in the pipe is 100 kNm^{-2} at A and 20 kNm^{-2} at B, both measured above atmosphere. The flow is $4 \text{ m}^3\text{min}^{-1}$ and is in direction A to B. Find the loss of energy, expressed as a head of water, between points A and B.

Solution

$$P_A = 100 \text{ kNm}^{-2}; \quad P_B = 20 \text{ kNm}^{-2}; \quad \text{and} \quad Q = 4 \text{ m}^3\text{min}^{-1} = 0.0667 \text{ m}^3\text{s}^{-1}$$

$$v_A = Q/A_A = 0.0667/[\frac{1}{4}\pi(0.16)^2] = 3.316 \text{ ms}^{-1}$$

$$v_B = Q/A_B = 0.0667/[\frac{1}{4}\pi(0.08)^2] = 13.27 \text{ ms}^{-1}$$

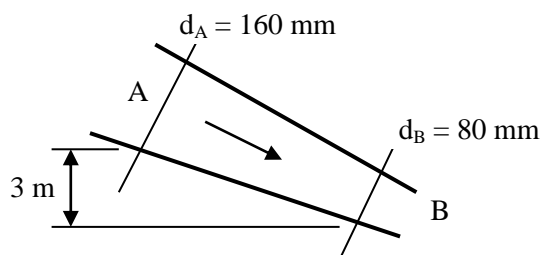


Fig. 5.9

$$z_A + \frac{P_A}{\rho g} + \frac{v_A^2}{2g} = z_B + \frac{P_B}{\rho g} + \frac{v_B^2}{2g} + H_L$$

$$3 + \frac{100 \times 10^3}{10^3 \times 9.81} + \frac{(3.32)^2}{2 \times 9.81} = 0 + \frac{20 \times 10^3}{10^3 \times 9.81} + \frac{(13.27)^2}{2 \times 9.81} + H_L$$

$$\therefore 3 + 10.2 + 0.5618 = 0 + 2.04 + 8.975 + H_L$$

$$H_L = 13.762 - 11.015 = \mathbf{2.747 \text{ m}}$$

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QUESTIONS AND ANSWERS

CHAPTER ONE

- 1.1 What will be (a) the gauge pressure, (b) the absolute pressure of water at a depth of 12m below the free surface? Assume the density of water to be 1000 kgm^{-3} and the atmospheric pressure 101kNm^{-2} . (Answer: 117.72 kNm^{-2} ; 218.72 kNm^{-2})
- 1.2 A cylinder contains 0.35m^3 of air at 50°C and 2.76 bar. The air is compressed to 0.071m^3 . Assuming isentropic condition, what is the final pressure and temperature, and what is the bulk modulus of elasticity ($\gamma = 1.4$)? (Answer: 25.75 bar; 338°C ; 3.6 MNm^{-2}).
- 1.3 The pressure of nitrogen gas at 27°C is 1 bar. If the gas constant is $0.287 \text{ kJkg}^{-1}\text{K}^{-1}$, calculate: (i) specific volume, (ii) specific weight, and (iii) specific gravity. (Answer: $0.861 \text{ m}^3\text{kg}^{-1}$; 11.39 Nm^{-3} ; 0.001161).
- 1.4 The density of oil at 20°C is 850 kgm^{-3} . Find its relative density and kinematic viscosity if the dynamic viscosity is $0.005 \text{ kgm}^{-1}\text{s}^{-1}$.
- 1.5 If the velocity distribution of a viscous liquid ($\mu = 0.9 \text{ Nsm}^{-2}$) over a fixed boundary is given by $u = 0.68y - y^2$ in which u is the velocity in ms^{-1} at a distance y metres above the boundary surface, determine the shear stress at the surface and at $y = 0.34 \text{ m}$.
- 1.6 At a depth of 8.5 km in the ocean the pressure is 90 MNm^{-2} . The specific weight of the sea water at the surface is 10.2 kNm^{-2} and its average bulk modulus is 2.4 GNm^{-2} . Determine: (a) the change in specific volume in m^3kg^{-1} , (b) the specific volume in m^3N^{-1} , and the specific weight of sea water at 8.5 km depth. (Answer: $3.606 \times 10^{-5} \text{ m}^3\text{kg}^{-1}$; $9.254 \times 10^{-4} \text{ m}^3\text{kg}^{-1}$; 10.6 kNm^{-3}).
- 1.7 Explain why the viscosity of a liquid decreases while that of a gas increases with an increase of temperature.
- 1.8 The following data refer to a liquid under shearing action at a constant temperature. Plot the graph of shear stress against velocity gradient and determine its dynamic viscosity.

du/dy (rad/s)	0	0.2	0.4	0.6	0.8
τ (N/m^2)	0	1.0	1.9	3.1	4.0

- 1.9 Calculate the capillary effect in mm in a glass tube of 6 mm diameter when immersed in (i) water and (ii) mercury, both liquids being at 20°C . Assume σ to be 0.073 Nm^{-1} for water and 0.5 Nm^{-1} for

mercury. The contact angles for water and mercury are 0° and 130° respectively. (Answer: 4.7 mm; – 1.6 mm).

- 1.10 Calculate the internal pressure of a 25 mm diameter soap bubble if the surface tension in the soap film is 0.5 Nm^{-1} .
- 1.11 A 0.6 mm diameter glass tube is inserted into water at 20°C and contact angle of 0° in a cup. Determine the capillary rise of water in the tube if the surface tension of water is 0.073 N/m . (Answer: 0.05 m).
- 1.12 A 1.9 mm diameter tube is inserted into an unknown liquid whose relative density is 0.96, and it is observed that the liquid rises 5 mm in the tube, making a contact angle of 15° . Determine the surface tension of the liquid.
- 1.13 Nutrients dissolved in water are carried to upper parts of plants by tiny tubes partly because of the capillary effect. Determine how high the water solution will rise in a tree in a 0.005 mm diameter tube as a result of the capillary effect. Treat the solution as water at 20°C with a contact angle of 15° . (Answer: 5.75 m).

CHAPTER TWO

- 2.1 Calculate the pressure in the ocean at a depth of 2000m assuming that salt water is incompressible with a constant density of 1002 kgm^{-3} . (Answer: 19.66 MNm^{-2})
- 2.2 Determine the pressure in N/m^2 at (a) a depth of 6 m below the free surface of a body of water and (b) at a depth of 9 m below the free surface of a body of oil of relative density 0.75. (Answer: 58.86 kNm^{-2} ; 66.22 kNm^{-2}).
- 2.3 What depth of oil, specific gravity 0.8, produces a pressure of 120 kNm^{-2} ? What would be the corresponding depth of water? (Answer: 15.3 m; 12.2 m)
- 2.4 A mass of 50 kg acts on a piston of area 100 cm^2 . What is the intensity of pressure on the water in contact with the underside of the piston, if the piston is in equilibrium? (Answer: 49.05 kNm^{-2})
- 2.5 A manometer connected to a pipe in which a fluid is flowing indicates a negative gauge pressure head of 50mm of mercury. What is the absolute pressure in the pipe in Nm^{-1} , if the atmospheric pressure is 1 bar and density of mercury is 13600 kgm^{-3} ? (Answer: 93.3 kN/m^2).
- 2.6 What is the gauge pressure and the absolute pressure of the air in Fig. Q2.6, if the barometric pressure is 780mm of mercury and the liquid is (a) water of density 1000 kgm^{-3} ; (b) oil of specific weight 7500 Nm^{-3} . (Answer: -4.9 kNm^{-2} , 99.2 kNm^{-2} ; -3.75 kNm^{-2} , 100.3 kNm^{-2}).

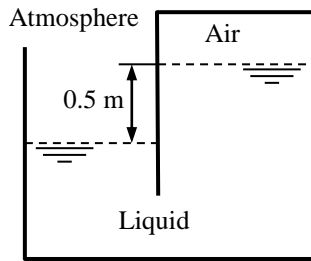


Fig. Q2.6

- 2.7 An open tank contains oil of specific gravity 0.75 on top of water. If the depth of oil is 2m and the depth of water 3m, calculate the gauge and absolute pressures at the bottom of the tank when the atmospheric pressure is 1 bar. (Answer: 44.15 kNm^{-2} ; 144.15 kNm^{-2}).

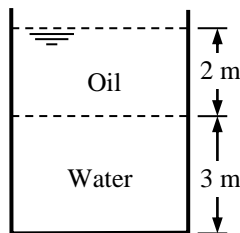


Fig. Q2.7

- 2.8 A closed tank contains 0.75 m of mercury (13.6 relative density), 3 m of water, 4.5 m of oil of relative density 0.6 and there is an air space above the oil. If the gauge pressure at the bottom of the tank is 300 kNm^{-2} , what is the pressure of the air at the top of the tank? (Answer: 144 kNm^{-2}).
- 2.9 Explain what is meant by stratified fluid. A pressure transducer is used to measure the hydrostatic pressure on the sea bed in a tidal estuary. The water in the estuary is stratified at the point where the measurement is taken, with fresh water of density 1000 kgm^{-3} overlying saline water of density 1025 kgm^{-3} . Water sampling shows that the fresh water extends from the water surface to a depth of 2.7 m. If the transducer indicates a gauge pressure of 69.73 kNm^{-2} , how thick is the layer of saline water? (Answer: 4.3 m).
- 2.10 The liquid in piezometer tube stand at 0.5 m above point 'A' in the pipe line, what is the gauge pressure at 'A' in kNm^{-2} if the liquid is (i) water, and (ii) mercury of specific gravity of 13.6. (Answer: 4.91 kNm^{-2} ; 6.67 kNm^{-2}).
- 2.11 A mercury U-tube manometer is used to measure the difference in pressure across a flow meter in a horizontal pipe, the water from the pipe being in contact with mercury in the manometer. The meter determines the flow rate of water according to the relation $Q = 0.26 \sqrt{h}$ where Q is the flow rate in m^3s^{-1} and h is the difference in head across the meter in metres of water (Answer: 0.533 m).

- 2.12 A U-tube manometer is arranged as shown in Fig. 2.7, to measure the pressure difference between two pipelines A and B, both carrying water of density 1000kg/m^3 . The density of manometric fluid is 13600kgm^{-3} . Calculate the pressure difference if h_1 is 80 cm, h_2 is 60 cm and h is 75 cm. (Answer: 98.1 kNm^{-2}).
- 2.13 An inverted U-tube of the form shown in Fig. 2.8 is used to measure the pressure difference between two points A and B. Fluid A is water and the top of manometer is filled with oil of relative density 0.75. If h_1 is 0.25 m, h_2 is 0.3 m and h is 0.2 m, calculate the pressure difference (a) when fluid B is liquid of relative density 0.8, and (b) when fluid B is water. (c) What will be pressure difference in (a) and (b) when the top of the manometer is filled with air instead of oil? (Answer: 1.57 kNm^{-2} ; 981 Nm^{-2} ; 98.1 Nm^{-2} ; -491 Nm^{-2}).

CHAPTER THREE

- 3.1 A tank is 180 cm long, 90 cm wide at the top, tapering to 30 cm wide at the base. If the tank is completely filled with water, calculate (a) the total weight of water in the tank, (b) the total force exerted by the water on the bottom, (c) the total force exerted on one end.
- 3.2 A rectangular tank of 1.5 m square base is filled with oil of density 917.5 kgm^{-3} to a depth of 2 m. Find (i) the resultant force on one of the vertical sides of the tank, (ii) the depth of the centre of pressure from the oil surface, and (iii) the total force exerted by the oil on the base of the tank. (Answer: 27.0 kN; 1.333 m; 40.5 kN).
- 3.3 A tank contains both oil and water. The oil has a depth of 1.5 m and a relative density of 0.8. It floats on top of the water with which it does not mix. The water has a depth of 2.0 m and the base of the tank is 3.0 m by 1.8 m. If the tank is open at the top to the atmosphere, calculate (a) the total weight of the contents of the tank; (b) the total pressure at the base of the tank (c) the total force on the 3.0 m long side of the tank. (Answer: 169.5 kN; 31.4 kNm^{-2} ; 156 kNm^{-2}).
- 3.4 A horizontal culvert with a trapezoidal section 150 cm wide at the top, 90 cm wide at the bottom and 1.8 m high, with sides inclined equally to the vertical, connects at one end to a reservoir in which the water surface is level with the top of the culvert and is closed at the other end by a vertical bulkhead fixed by lugs at the four corners. Calculate: (a) the total force due to the water on the bulkhead and (b) the depth of the centre of pressure.
- 3.5 A rectangular opening in the sloping side of a fresh water reservoir is 1.8m by 1.2m, the 1.2m side being horizontal (Fig. Q3.5). The opening is filled by a door of uniform thickness, hinged at the top and the door is kept closed by its own weight. If the gate commences to open when the water level is

0.2m above the top of the gate, calculate: (a) the total force on the door; (b) the depth at which the total force acts; and (c) the weight of the door. (Answer: 20.8 kN; 1.186 m; 52.5 kN).

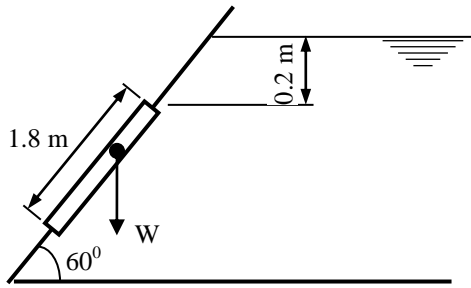


Fig. Q3.5

3.6 A dam has its water face in the shape of a circular arc as shown in Fig. Q3.6. Calculate: (i) the resultant force in kN on 10m wide face when the depth of water is 12m; and (ii) the inclination of the resultant force to the horizontal. (Answer: 8617 kN; 35° to the horizontal).

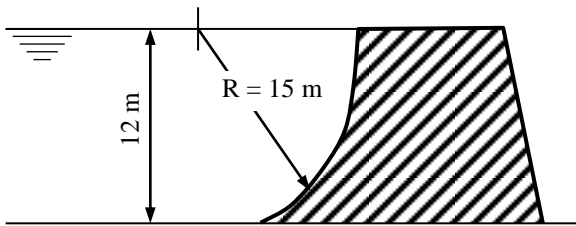


Fig. Q3.6

3.7 The dam in Fig. Q3.7 has a curved face being part of a 40 m radius circle. The dam holds back water to a depth of 35 m. Calculate the magnitude and direction of the resultant hydrostatic force per meter length. (Answer: 7834 kNm⁻¹; 40° to the horizontal).

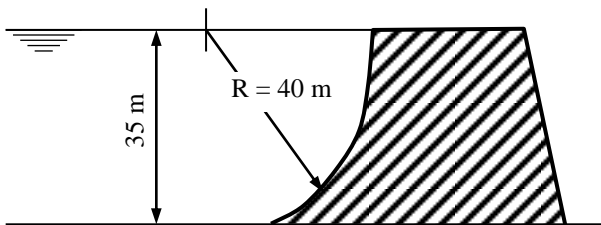


Fig. Q3.7

- 3.8 A gate which is a quarter of a circle of radius 4.0 m holds back 2.0 m height of fresh water as shown in Fig. Q3.8. Calculate the magnitude and direction (angle makes with the horizontal) of the resultant hydrostatic force for 1 m length of the gate. (Answer: 52.04 kNm^{-1} ; 68° to the horizontal).

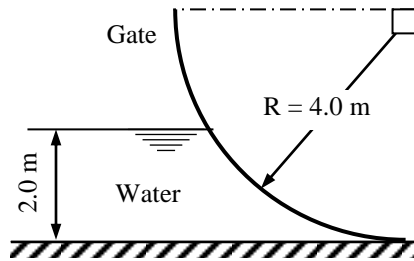


Fig. Q3.8

CHAPTER FOUR

- 4.1 A rectangular block of wood floats in water with 50 mm projecting above the water surface. When placed in glycerin of relative density 1.35, the block projects 75 mm above the surface of glycerin. Determine the relative density of the wood. (Answer: 0.658).
- 4.2 A cylindrical buoy 1.8m in diameter and 1.2m high float in sea water of density 1025 kg/m^3 . If the mass of the buoy is 900kg determine: (i) the distance of centre of buoyancy from the centre of gravity of the buoy; (ii) the distance of metacentre from the centre of gravity of the buoy. (Answer: 0.428 m)
- 4.3 A floating crane has a weight of 190 kN and float in sea water of density 1050 kgm^{-3} . If the base of the crane is 4 m square and 2.5 m deep, what is the maximum weight that can be lifted by the crane when the top of the base is just level with water. (Answer: 222 kN).
- 4.4 A cylinder bouy floats in sea water with an iron chain fastened to the base to keep its axis vertical so that it is two-thirds submerged (Fig. Q4.4). The buoy is 0.8 m in diameter and 2 m in height and is fabricated from iron plate 12 mm thick. If the density of iron, ρ_{iron} is 7700 kg/m^3 and of sea water, ρ_{sea} is 1030 kg/m^3 , calculate: (i) the volume of iron plate that formed the cylindrical buoy; (ii) the weight of buoy, W; (iii) the downward pull 'P' by the chain; and (iv) the volume and the weight of the chain. (Answer: 0.0708 m^3 ; 5.35 kN; 1.43 kN; 0.02184 m^3 ; 1.65 kN).

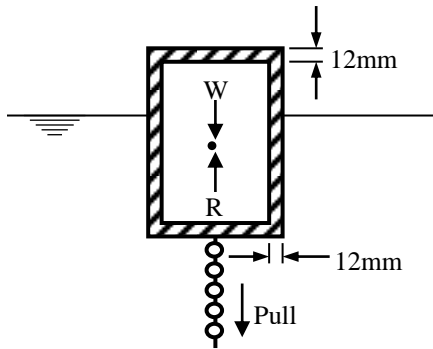


Fig. Q4.4

CHAPTER FIVE

- 5.1 If 945 dm^3 of water are discharged from a vessel in 15 seconds, find the rate of discharge in m^3s^{-1} . If the discharge took place through an opening 120 mm diameter, determine the velocity of discharge. (Answer: $0.063 \text{ m}^3\text{s}^{-1}$; 5.57 ms^{-1}).
- 5.2 A piston of 72 mm diameter slides concentrically in a fixed cylinder of 80 mm diameter. The cylinder is filled with oil and when the piston moves into the cylinder the oil flows through the annular gap surrounding the piston. If the velocity of the piston is 12 cms^{-1} relative to the cylinder, what is the velocity of flow through the gap? (0.512 ms^{-1}).
- 5.3 In a vertical pipe line conveying water, pressure gauges are inserted at A and B where the diameters are 150 mm and 75 mm respectively. The point B is 2.4 m below A and when the rate of flow down the pipe is $0.02 \text{ m}^3\text{s}^{-1}$, the pressure at B is 12 kNm^{-2} greater than at A. Assuming the losses in the pipe between A and B can be expressed as $K \cdot v^2 / 2g$, where v is the velocity at A, find the value of K. (Answer: 3.03).
- 5.4 A pump delivers water from a lower to a higher reservoir. The difference in elevation between the reservoirs is 10 m. The pump provides an energy head of 11 m and the frictional head losses are 0.7 m. If the pipe diameter is 300 mm, calculate the discharge. (Answer: $0.171 \text{ m}^3\text{s}^{-1}$).
- 5.5 Fig. Q5.4 shows part of the lubrication system of an engine. At 'A' the pressure is 80 kNm^{-2} . The diameter of the suction pipe is twice that of the delivery pipe. The total head lost in the system is 440 mm of oil and point B is 0.8 m above A. the pump supplies a head of 11.3 m of oil and the oil has a density of 850 kgm^{-3} . Determine the velocity of flow in the suction pipe. (Answer: 0.775 ms^{-1}).

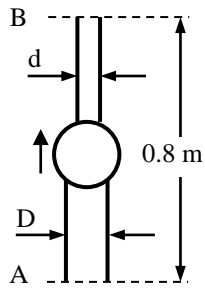


Fig. Q5.5

- 5.6 The suction pipe of a pump rises at a slope of 1 vertical in 5 along the pipe and water passes through it at 1.8 ms^{-1} . If dissolved air is released when pressure falls to more than 70 kNm^{-2} below atmospheric pressure, find the greatest practicable length of pipe, neglecting friction. Assume that the water in the sump is at rest. (Answer: 35.4 m).
- 5.7 A pipe 300 m long tapers from 1.2 m diameter to 0.6 m diameter at its lower end and slopes downward at 1 in 100. The pressure at the upper end is 69 kNm^{-2} . Neglecting friction losses, find the pressure at the lower end when the rate of flow is $5.5 \text{ m}^3/\text{min}$. (Answer: 98.5 kNm^{-2}).
- 5.8 The pressure in an hydraulic accumulator is 1250 kNm^{-2} . If 225 dm^3 of water are pumped into the cylinder in 5 min., what is the work done and the power required? (Answer: 281 kJ; 937 W).
- 5.9 A pump draws water through a 150 mm diameter pipe from a reservoir whose surface level is at datum level and discharges it through a 100 mm pipe to another reservoir whose surface level is 72 m above datum level. The pump is situated 6 m below datum. The loss of head in the 150 mm suction pipe is 3 times the velocity head in the 150 mm pipe and the loss of head in the 100 mm delivery pipe is 20 times the velocity head in the 100 mm pipe. Calculate the power out put of the pump and the pressure heads above atmospheric pressure at inlet and outlet of the pump when the discharge is (a) $0.91 \text{ m}^3/\text{min}$, and (b) $2.73 \text{ m}^3/\text{min}$. (Answer: 11.29 kW; 5.96 m; 81.84 m; 47.7 kW; 5.66 m; 112.53 m).
- 5.10 A storage reservoir supplies water to a pressure turbine under a head of 20 m. When the turbine draws $500 \text{ dm}^3\text{s}^{-1}$ of water the head loss in the 300 mm diameter supply line amounts to 2.5 m. Determine the pressure intensity at the entrance to the turbine. If negative pressure of 30 kNm^{-2} exists at the 600 mm diameter section of the draft tube 1.5 m below the supply line, estimate the energy absorbed by the turbine in kW neglecting all frictional losses between the entrance and exit of the turbine. Hence find the output of the turbine assuming an efficiency of 85%. (Answer: 146.7 kNm^{-2} ; 107.4 kW; 91.31 kW).